AQA Maths Decision 2

Past Paper Pack

2006-2015

General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Wednesday 18 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 3 and 4 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 Five trainers, Ali, Bo, Chas, Dee and Eve, held an initial training session with each of four teams over an assault course. The completion times in minutes are recorded below.

	Ali	Bo	Chas	Dee	Eve
Team 1	16	19	18	25	24
Team 2	22	21	20	26	25
Team 3	21	22	23	21	24
Team 4	20	21	21	23	20

Each of the four teams is to be allocated a trainer and the overall time for the four teams is to be minimised. No trainer can train more than one team.

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four trainers should be allocated to which team. State the minimum total training time for the four teams using this matching. *(8 marks)*

2 A manufacturing company is planning to build three new machines, A, B and C, at the rate of one per month. The order in which they are built is a matter of choice, but the profits will vary according to the number of workers available and the suppliers' costs. The expected profits in thousands of pounds are given in the table.

Month	**Already built**	**Profit (in units of £1000)**		
		A	B	C
1	—	52	47	48
2	A	—	58	54
	B	70	—	54
	C	68	63	—
3	A **and** B	—	—	64
	A **and** C	—	67	—
	B **and** C	69	—	—

(a) Draw a labelled network such that the most profitable order of manufacture corresponds to the longest path within that network. *(2 marks)*

(b) Use dynamic programming to determine the order of manufacture that **maximises** the total profit, and state this maximum profit. *(7 marks)*

3 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

A building project is to be undertaken. The table shows the activities involved.

Activity	Immediate Predecessors	Duration (days)	Number of Workers Required
A	—	2	3
B	*A*	4	2
C	*A*	6	1
D	*B*, *C*	8	3
E	*C*	3	2
F	*D*	2	2
G	*D*, *E*	4	2
H	*D*, *E*	6	1
I	*F*, *G*, *H*	2	3

(a) Complete the activity network for the project on **Figure 1**. *(3 marks)*

(b) Find the earliest start time for each activity. *(2 marks)*

(c) Find the latest finish time for each activity. *(2 marks)*

(d) Find the critical path and state the minimum time for completion. *(2 marks)*

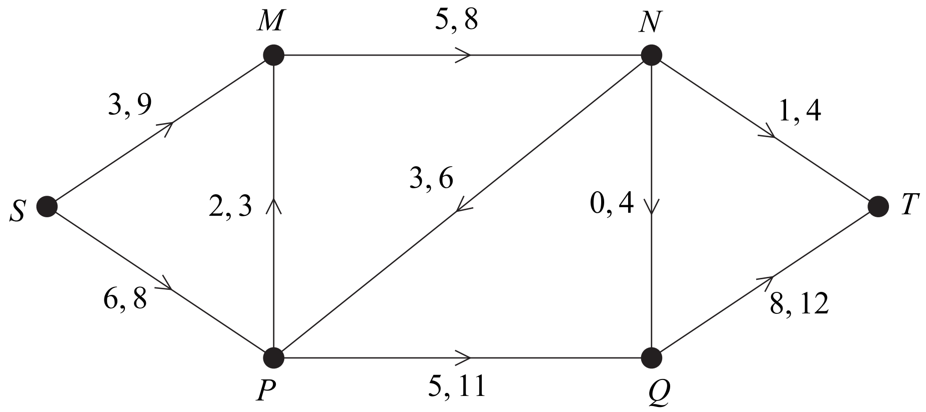
(e) State the float time for each non-critical activity. *(2 marks)*

(f) Given that each activity starts as early as possible, draw a resource histogram for the project on **Figure 2**. *(4 marks)*

(g) There are only 3 workers available at any time. Use resource levelling to explain why the project will overrun and state the minimum extra time required. *(3 marks)*

4 [**Figures 3**, **4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes, with the lower and upper capacities for each pipe in litres per second.



(a) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 10 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges MN, PQ, NP and NT. *(4 marks)*

(b) (i) Taking your answer from part (a) as an initial flow, use flow augmentation on **Figure 4** to find the maximum flow from S to T. *(6 marks)*

(ii) State the value of the maximum flow and illustrate this flow on **Figure 5**. *(2 marks)*

(c) Find a cut with capacity equal to that of the maximum flow. *(2 marks)*

5 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 3x + 2y + 4z$

subject to

$$x + 4y + 2z \leqslant 8$$
$$2x + 7y + 3z \leqslant 21$$
$$x \geqslant 0, y \geqslant 0, z \geqslant 0$$

(3 marks)

(b) Use the Simplex method to perform **one** iteration of your tableau for part (a), choosing a value in the z-column as pivot. *(3 marks)*

(c) (i) Perform one further iteration. *(5 marks)*

(ii) State whether or not this is the optimal solution, and give a reason for your answer. *(2 marks)*

6 Sam is playing a computer game in which he is trying to drive a car in different road conditions. He chooses a car and the computer decides the road conditions. The points scored by Sam are shown in the table.

Road Conditions

		C_1	C_2	C_3
	S_1	-2	2	4
Sam's Car	S_2	2	4	5
	S_3	5	1	2

Sam is trying to maximise his total points and the computer is trying to stop him.

(a) Explain why Sam should never choose S_1 and why the computer should not choose C_3. *(2 marks)*

(b) Find the play-safe strategies for the reduced 2 by 2 game for Sam and the computer, and hence show that this game does not have a stable solution. *(4 marks)*

(c) Sam uses random numbers to choose S_2 with probability p and S_3 with probability $1 - p$.

(i) Find expressions for the expected gain for Sam when the computer chooses each of its two remaining strategies. *(3 marks)*

(ii) Calculate the value of p for Sam to maximise his total points. *(2 marks)*

(iii) Hence find the expected points gain for Sam. *(1 mark)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Insert

Wednesday 18 January 2006 1.30 pm to 3.00 pm

Insert for use in **Questions 3 and 4.**

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

For use in Question 3

Figure 1 (for use in part (a))

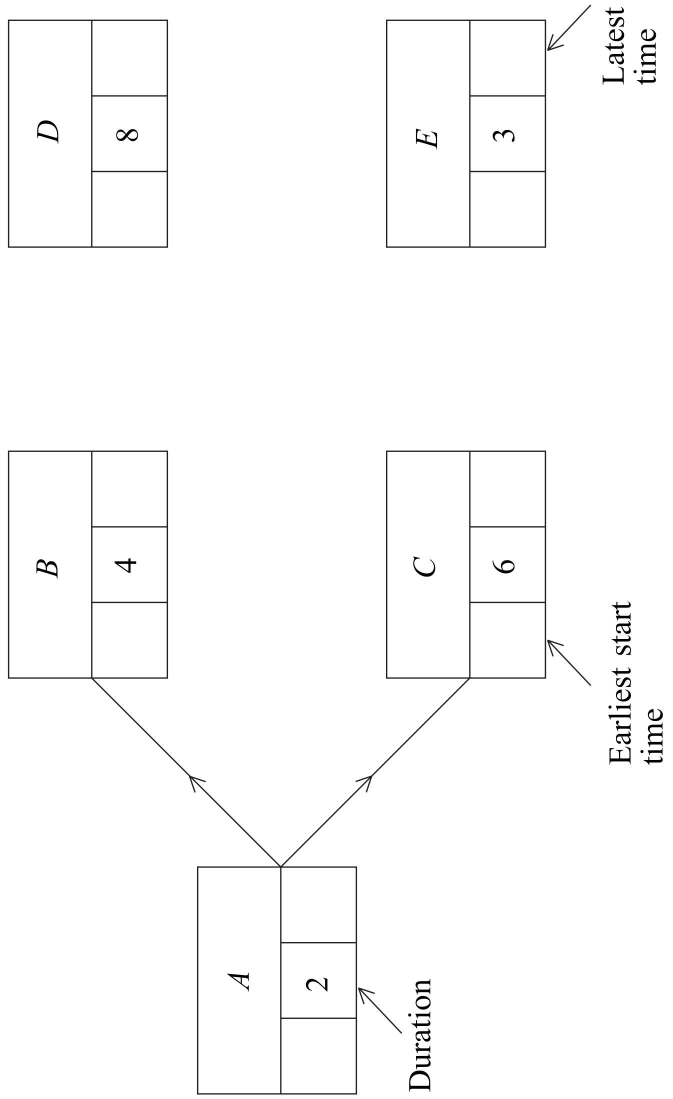
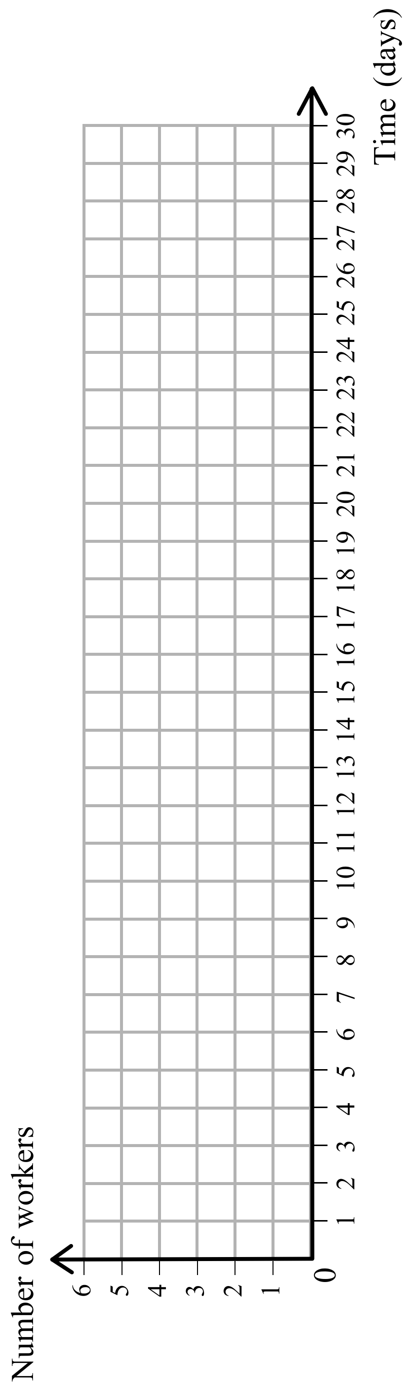


Figure 2 (for use in part (f))



For use in Question 4

Figure 3 (for use in part (a))

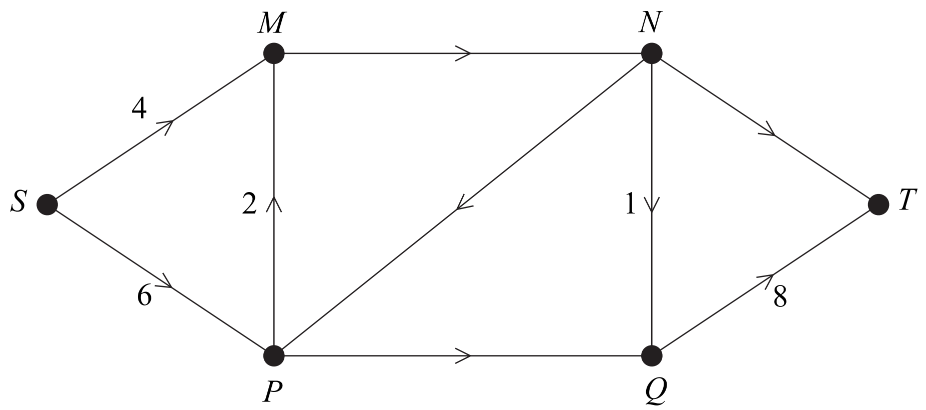
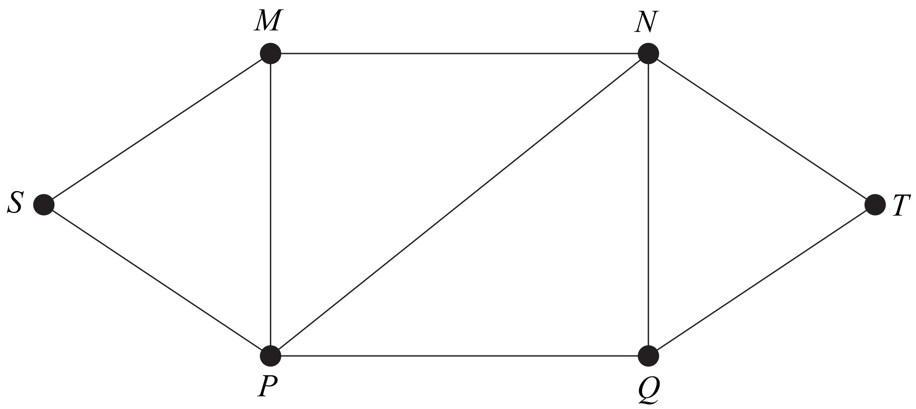
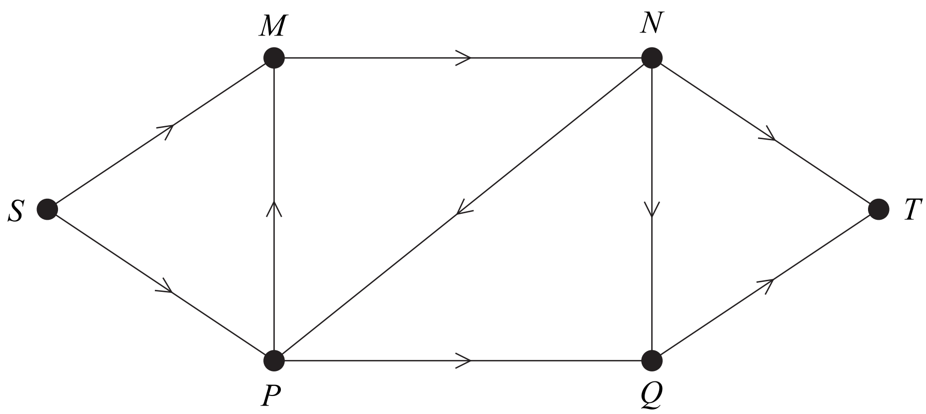


Figure 4 (for use in part (b)(i))



Route	Extra flow

Figure 5 (for use in part (b)(ii))



General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Thursday 8 June 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
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You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
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Information
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- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

A construction project is to be undertaken. The table shows the activities involved.

Activity	**Immediate Predecessors**	**Duration (days)**
A	–	2
B	A	5
C	A	8
D	B	8
E	B	10
F	B	4
G	C, F	7
H	D, E	4
I	G, H	3

(a) Complete the activity network for the project on **Figure 1**. *(3 marks)*

(b) Find the earliest start time for each activity. *(2 marks)*

(c) Find the latest finish time for each activity. *(2 marks)*

(d) Find the critical path. *(1 mark)*

(e) State the float time for each non-critical activity. *(2 marks)*

(f) On **Figure 2**, draw a cascade diagram (Gantt chart) for the project, assuming each activity starts as **late** as possible. *(4 marks)*

2 Four of the five students Phil, Quin, Ros, Sue and Tim are to be chosen to make up a team for a mathematical relay race. The team will be asked four questions, one each on the topics A, B, C and D. A different member of the team will answer each question. Each member has to give the correct answer to the question before the next question is asked. The team with the least overall time wins.
The average times, in seconds, for each student in some practice questions are given below.

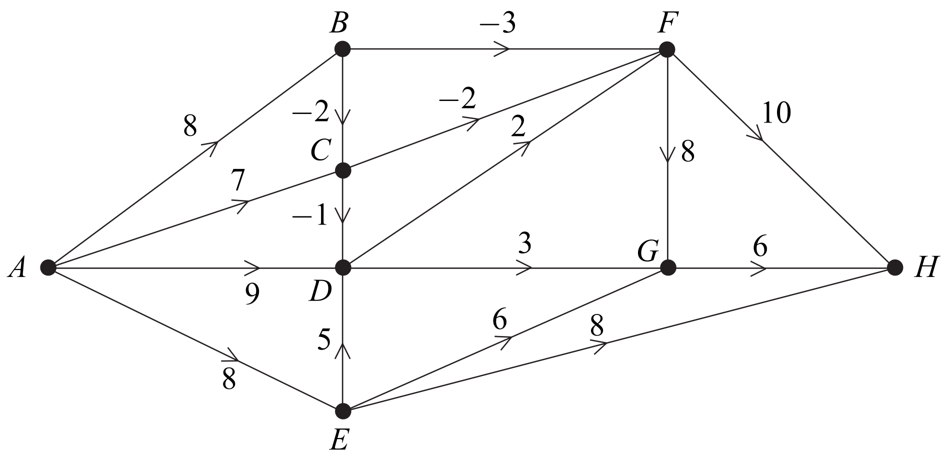
	Phil	**Quin**	**Ros**	**Sue**	**Tim**
Topic A	18	15	19	20	17
Topic B	23	24	22	25	23
Topic C	20	16	18	22	19
Topic D	21	17	18	23	20

(a) Modify the table of values by adding an extra row of values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first**, then rows, to decide which four students should be chosen for the team. State which student should be allocated to each topic and state the total time for the four students on the practice questions using this matching. *(8 marks)*

3 [**Figure 3**, printed on the insert, is provided for use in this question.]

The following network shows eight vertices. The number on each edge is the cost of travelling between the corresponding vertices. A negative number indicates a reduction by the amount shown.



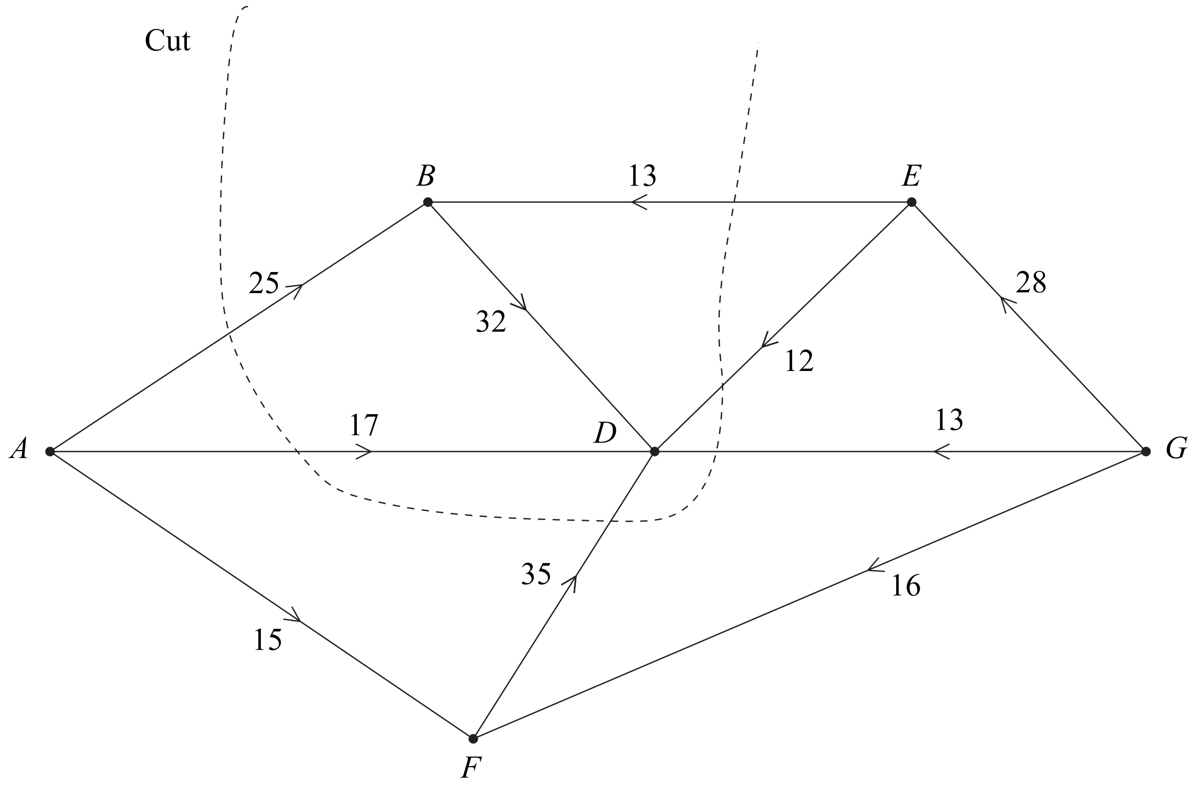
(a) Use dynamic programming to find the minimum cost of travelling from A to H. You may use **Figure 3** for your working. *(6 marks)*

(b) State the minimum cost and the possible routes corresponding to this minimum cost. *(3 marks)*

Turn over ▶

4 [**Figures 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows the routes along corridors from the playgrounds A and G to the assembly hall in a school. The number on each edge represents the maximum number of pupils that can travel along the corridor in one minute.



(a) State the vertex that represents the assembly hall. *(1 mark)*

(b) Find the value of the cut shown on the diagram. *(1 mark)*

(c) State the maximum flow along the routes ABD and GED. *(2 marks)*

(d) (i) Taking your answers to part (c) as the initial flow, use a labelling procedure on **Figure 4** to find the maximum flow through the network. *(6 marks)*

(ii) State the value of the maximum flow and, on **Figure 5**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Verify that your flow is a maximum flow by finding a cut of the same value. *(2 marks)*

(e) On a particular day, there is an obstruction allowing no more than 15 pupils per minute to pass through vertex E. State the maximum number of pupils that can move through the network per minute on this particular day. *(2 marks)*

5 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 4x + 9y$. The initial Simplex tableau is given below.

P	x	y	r	s	t	*value*
1	−4	−9	0	0	0	0
0	3	7	1	0	0	33
0	1	2	0	1	0	10
0	2	7	0	0	1	26

(a) Write down the **three** inequalities in x and y represented by this tableau. *(2 marks)*

(b) The Simplex method is to be used to solve this linear programming problem by initially choosing a value in the x-column as the pivot.

(i) Explain why the initial pivot has value 1. *(2 marks)*

(ii) Perform **two** iterations using the Simplex method. *(7 marks)*

(iii) Comment on how you know that the optimum solution has been achieved and state your final values of P, x and y. *(3 marks)*

6 Two people, Rowan and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rowan.

Colleen

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	−3	−4	1
Rowan	$\mathbf{R_2}$	1	5	−1
	$\mathbf{R_3}$	−2	−3	4

(a) Explain the meaning of the term 'zero-sum game'. *(1 mark)*

(b) Show that this game has no stable solution. *(3 marks)*

(c) Explain why Rowan should never play strategy R_1. *(1 mark)*

(d) (i) Find the optimal mixed strategy for Rowan. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

END OF QUESTIONS

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Candidate Signature									

General Certificate of Education
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MATHEMATICS **MD02**
Unit Decision 2

Insert

Thursday 8 June 2006 9.00 am to 10.30 am

Insert for use in **Questions 1, 3 and 4.**

Fill in the boxes at the top of this page.

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Turn over for Figure 1

Turn over ▶

For use in Question 1

Figure 1 (for use in parts (a), (b) and (c))

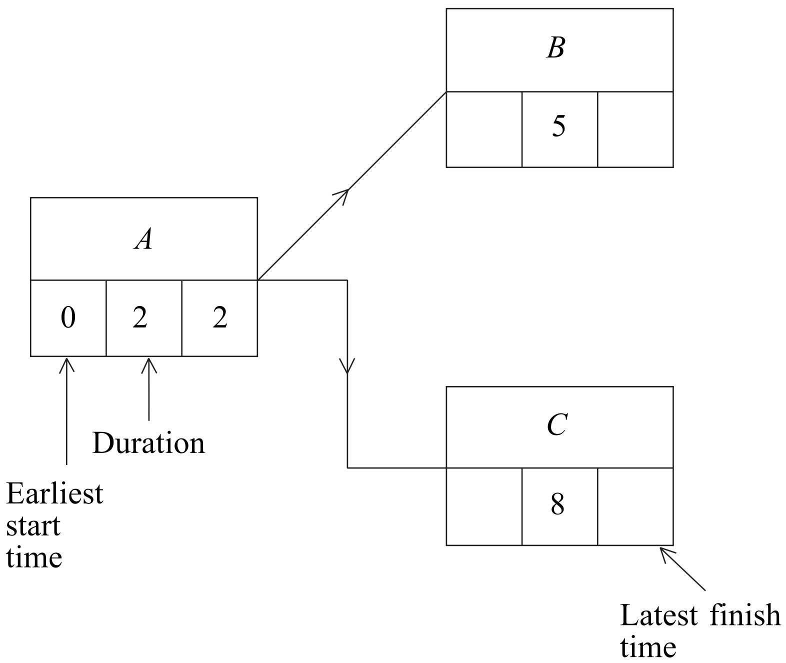


Figure 2 (for use in part (f))

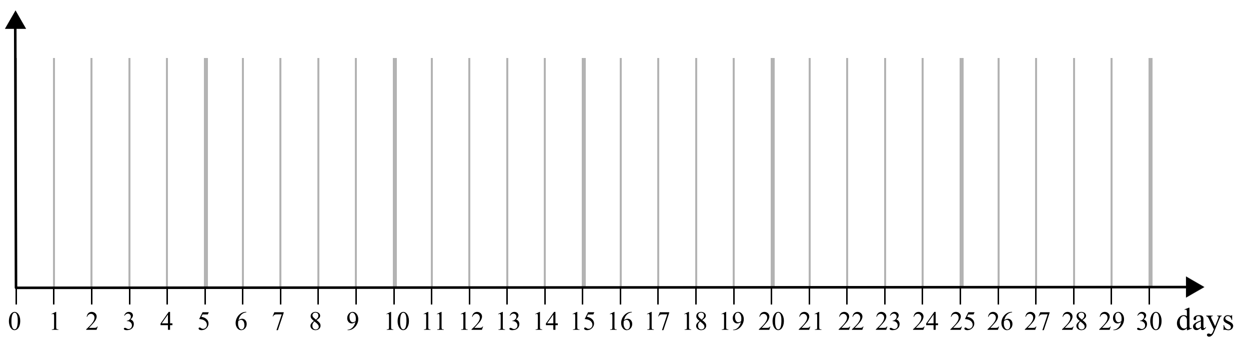
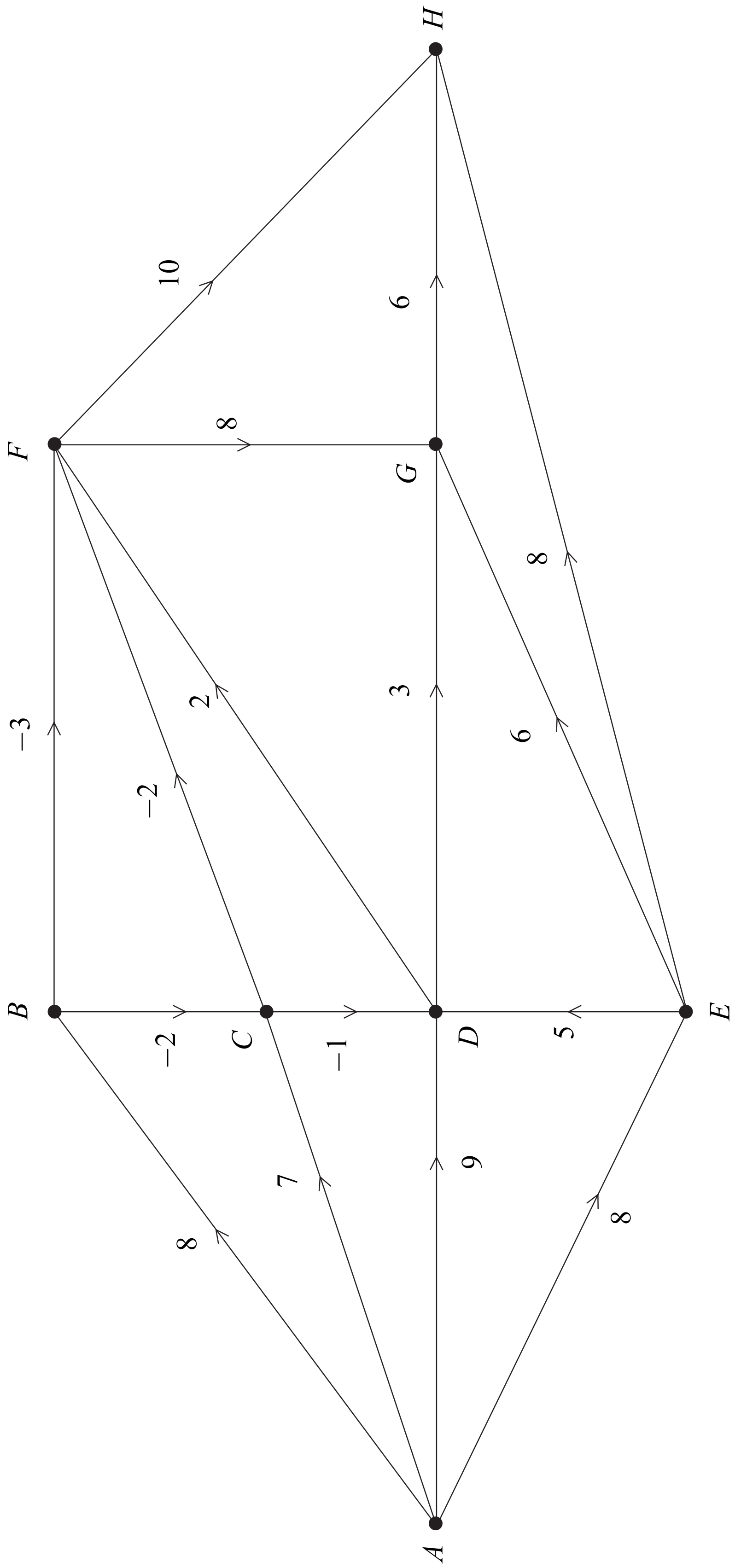
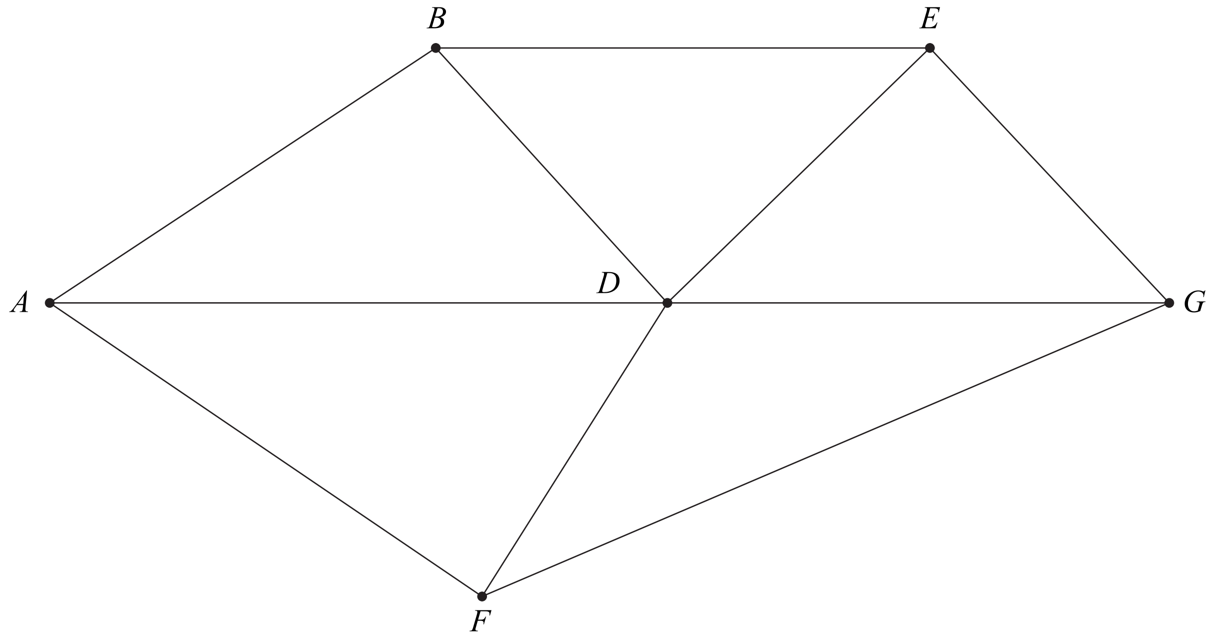


Figure 3 (for use in Question 3)



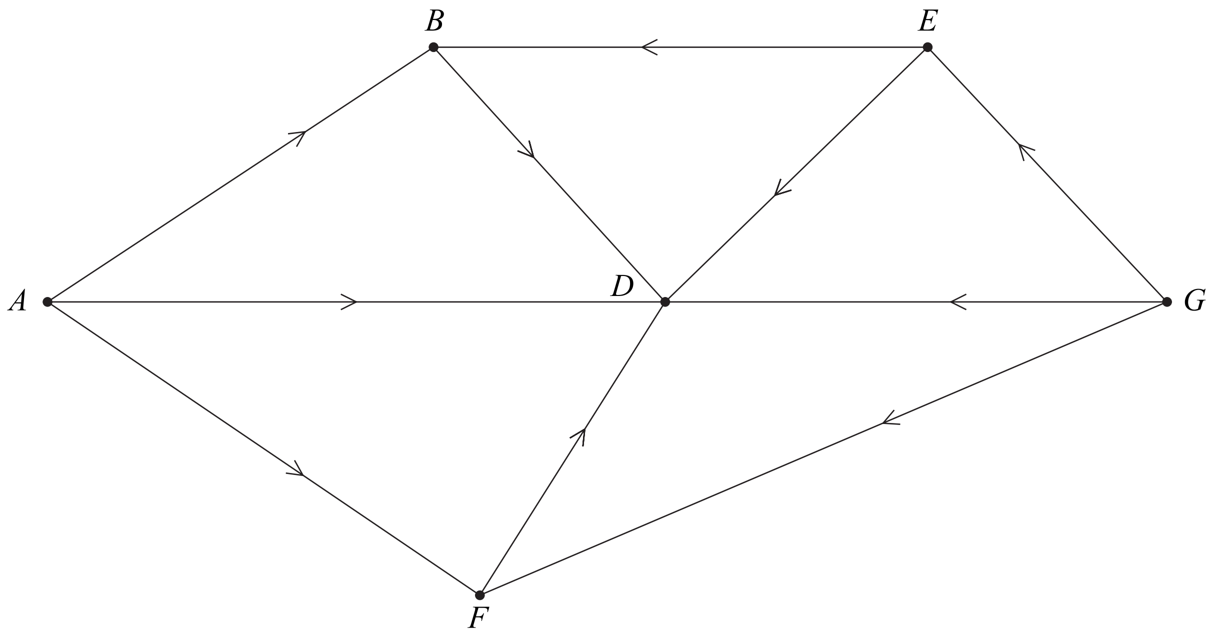
Turn over ▶

Figure 4 (for use in Question 4 part (d)(i))



Route	Flow
ABD	
GED	

Figure 5 (for use in Question 4 part (d)(ii))



General Certificate of Education
January 2007
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Wednesday 31 January 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 1 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

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Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in this question.]

A building project is to be undertaken. The table shows the activities involved.

Activity	Immediate Predecessors	Duration (weeks)
A	–	2
B	–	1
C	A	3
D	A, B	2
E	B	4
F	C	1
G	C, D, E	3
H	E	5
I	F, G	2
J	H, I	3

(a) Complete an activity network for the project on **Figure 1**. *(3 marks)*

(b) Find the earliest start time for each activity. *(2 marks)*

(c) Find the latest finish time for each activity. *(2 marks)*

(d) State the minimum completion time for the building project and identify the critical paths. *(4 marks)*

2 Five successful applicants received the following scores when matched against suitability criteria for five jobs in a company.

	Job 1	**Job 2**	**Job 3**	**Job 4**	**Job 5**
Alex	13	11	9	10	13
Bill	15	12	12	11	12
Cath	12	10	8	14	14
Don	11	12	13	14	10
Ed	12	14	14	13	14

It is intended to allocate each applicant to a different job so as to maximise the total score of the five applicants.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $15 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table from 15. Use the Hungarian algorithm to allocate the jobs to the applicants so that the total score is maximised. *(8 marks)*

(c) It is later discovered that Bill has already been allocated to Job 4. Decide how to alter the allocation of the other jobs so as to maximise the score now possible. *(3 marks)*

3 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 5x + 8y + 7z$

subject to

$$3x + 2y + z \leqslant 12$$
$$2x + 4y + 5z \leqslant 16$$
$$x \geqslant 0, y \geqslant 0, z \geqslant 0$$

(3 marks)

(b) The Simplex method is to be used by initially choosing a value in the y-column as a pivot.

(i) Explain why the initial pivot is 4. *(1 mark)*

(ii) Perform **two** iterations of your tableau from part (a) using the Simplex method. *(6 marks)*

(iii) State the values of P, x, y and z after your second iteration. *(2 marks)*

(iv) State, giving a reason, whether the maximum value of P has been achieved. *(1 mark)*

Turn over ▶

4 (a) Two people, Ros and Col, play a zero-sum game. The game is represented by the following pay-off matrix for Ros.

		Col		
	Strategy	**X**	**Y**	**Z**
Ros	**I**	−4	−3	0
	II	5	−2	2
	III	1	−1	3

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player and state the value of the game. *(2 marks)*

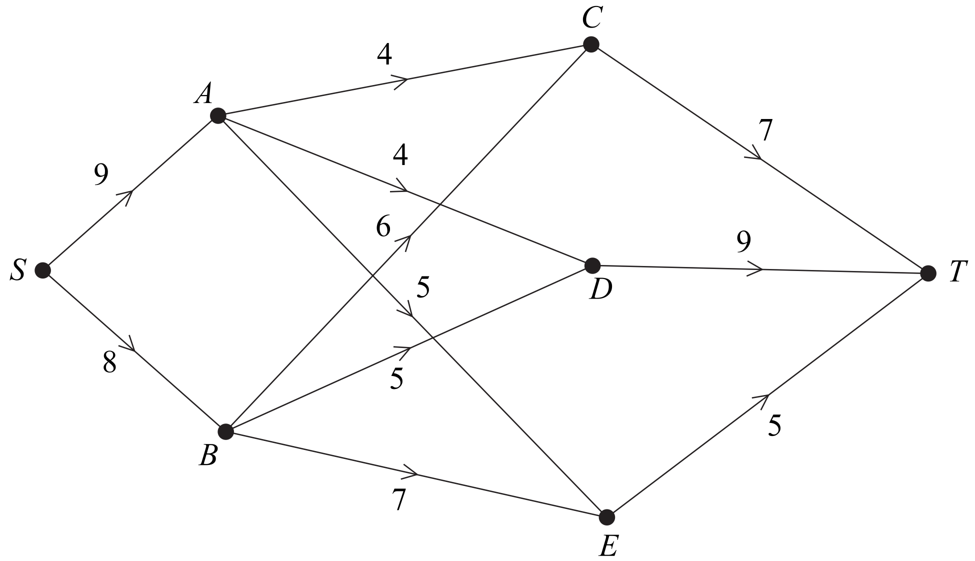
(b) Ros and Col play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros.

		Col		
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Ros	$\mathbf{R_1}$	3	2	1
	$\mathbf{R_2}$	−2	−1	2

(i) Find the optimal mixed strategy for Ros. *(7 marks)*

(ii) Calculate the value of the game. *(1 mark)*

5 A three-day journey is to be made from S to T, with overnight stops at the end of the first day at either A or B and at the end of the second day at one of the locations C, D or E. The network shows the number of hours of sunshine forecast for each day of the journey.



The optimal route, known as the maximin route, is that for which the least number of hours of sunshine during a day's journey is as large as possible.

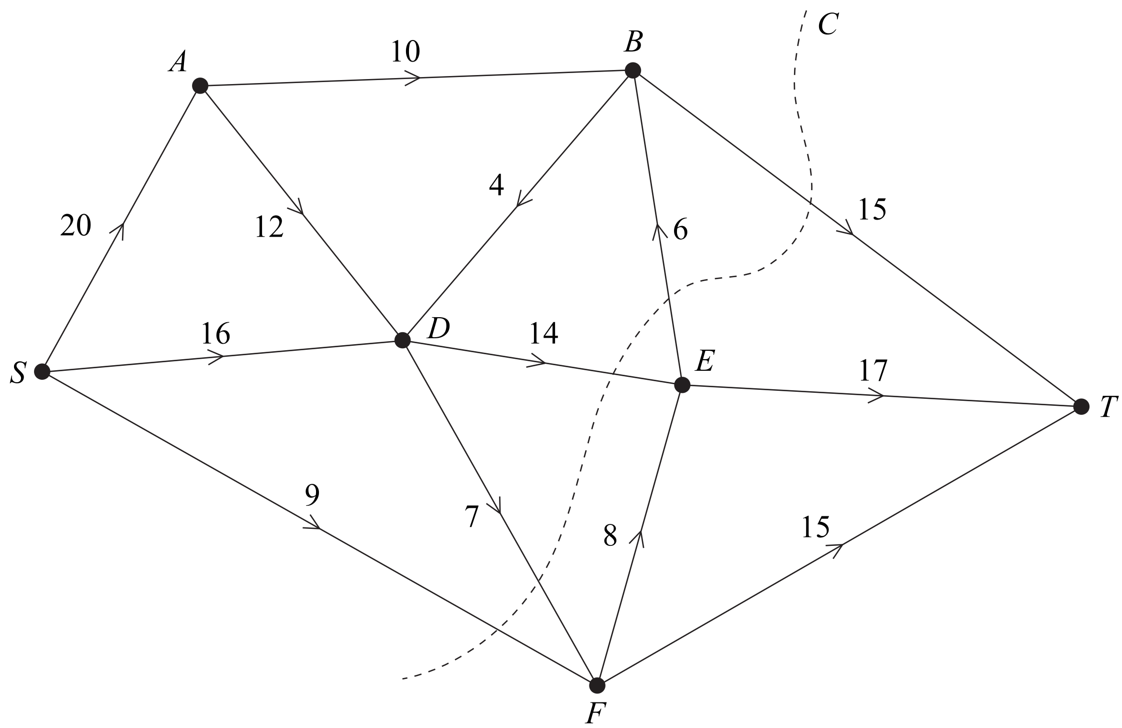
(a) Explain why the three-day route $SAET$ is better than $SACT$. *(2 marks)*

(b) Use dynamic programming to find the optimal (maximin) three-day route from S to T. *(8 marks)*

Turn over for the next question

6 [**Figures 2** and **3**, printed on the insert, are provided for use in this question.]

The diagram shows a network of pipelines through which oil can travel. The oil field is at S, the refinery is at T and the other vertices are intermediate stations. The weights on the edges show the capacities in millions of barrels per hour that can flow through each pipeline.



(a) (i) Find the value of the cut marked C on the diagram. *(1 mark)*

(ii) Hence make a deduction about the maximum flow of oil through the network. *(2 marks)*

(b) State the maximum possible flows along the routes $SABT$, $SDET$ and SFT. *(2 marks)*

(c) (i) Taking your answer to part (b) as the initial flow, use a labelling procedure on **Figure 2** to find the maximum flow from S to T. Record your routes and flows in the table provided and show the augmented flows on the network diagram. *(6 marks)*

(ii) State the value of the maximum flow, and, on **Figure 3**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Prove that your flow in part (c)(ii) is a maximum. *(2 marks)*

END OF QUESTIONS

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Candidate Signature										

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MATHEMATICS **MD02**
Unit Decision 2

Insert

Insert for use in **Questions 1 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 1)

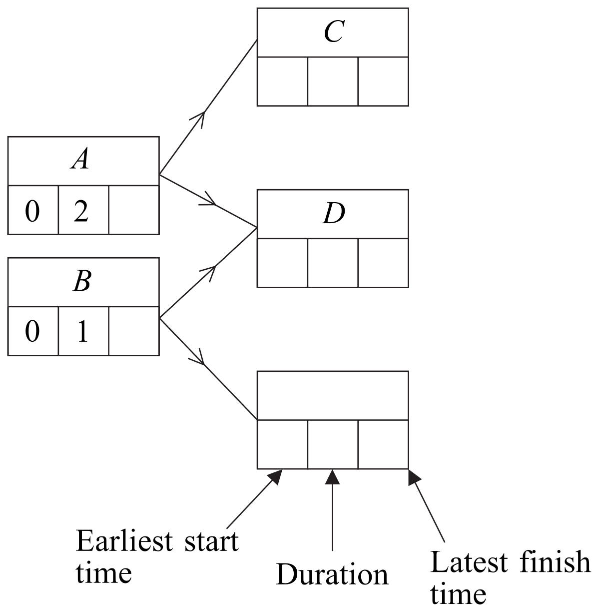
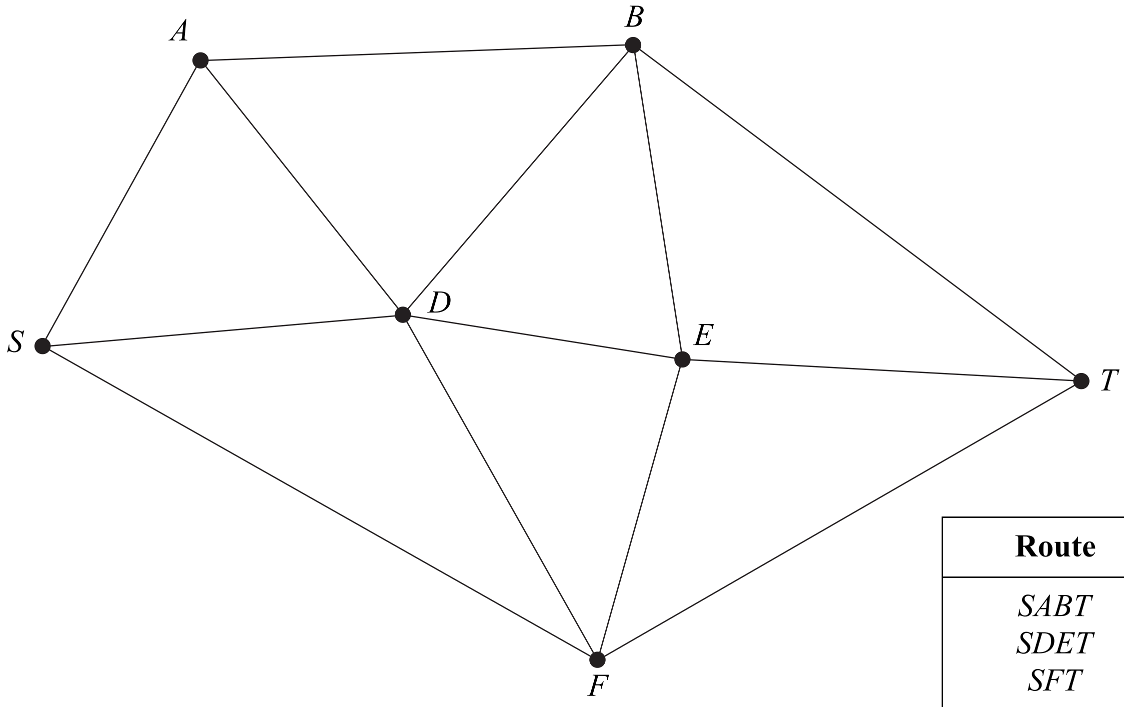
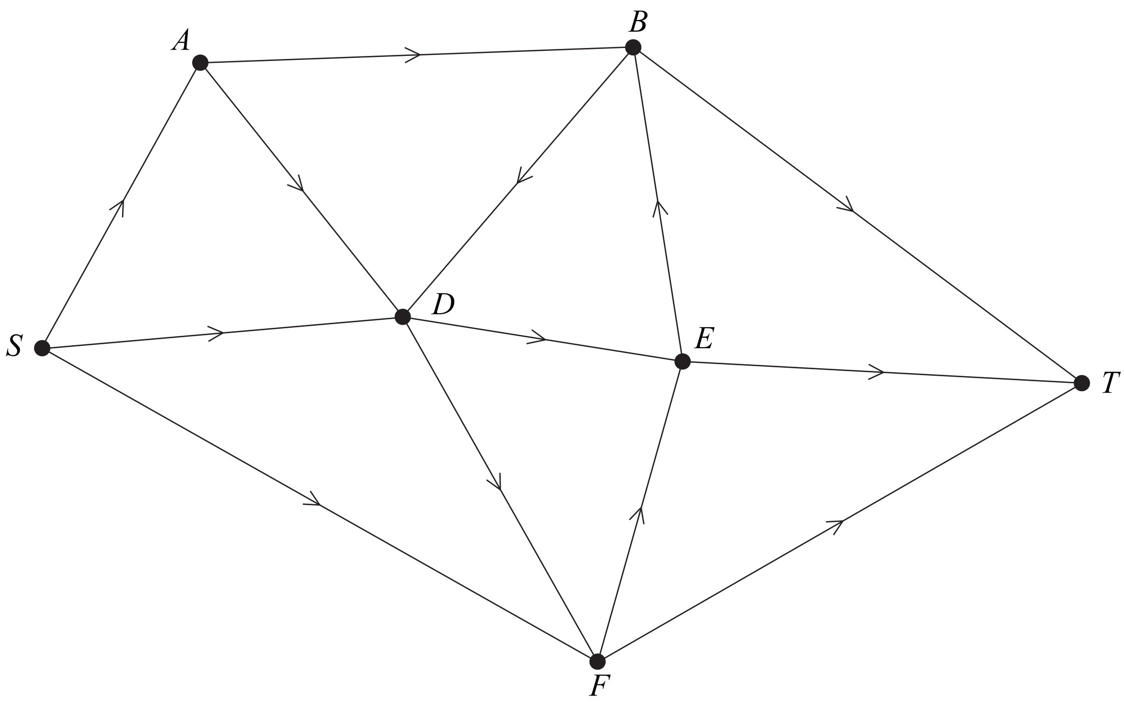


Figure 2 (for use in Question 6(c)(i))



Route	Flow
SABT *SDET* *SFT*	

Figure 3 (for use in Question 6(c)(ii))



General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

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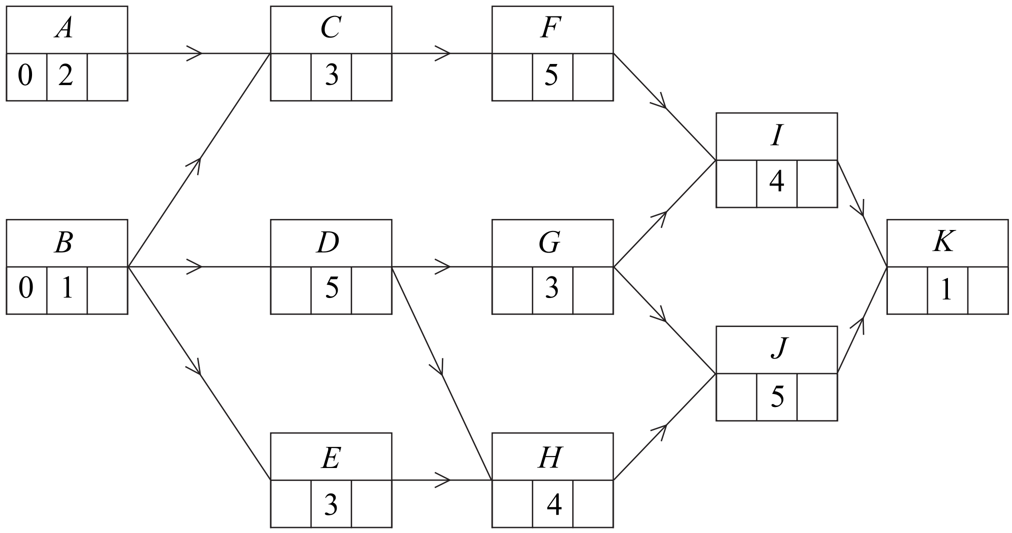
Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

The following diagram shows an activity diagram for a building project. The time needed for each activity is given in days.



(a) Complete the precedence table for the project on **Figure 1**. *(2 marks)*

(b) Find the earliest start times and latest finish times for each activity and insert their values on **Figure 2**. *(4 marks)*

(c) Find the critical path and state the minimum time for completion of the project. *(2 marks)*

(d) Find the activity with the greatest float time and state the value of its float time. *(2 marks)*

2 The daily costs, in pounds, for five managers A, B, C, D and E to travel to five different centres are recorded in the table below.

	A	B	C	D	E
Centre 1	10	11	8	12	5
Centre 2	11	5	11	6	7
Centre 3	12	8	7	11	4
Centre 4	10	9	14	10	6
Centre 5	9	9	7	8	9

Using the Hungarian algorithm, each of the five managers is to be allocated to a different centre so that the overall total travel cost is minimised.

(a) By reducing the **rows first** and then the columns, show that the new table of values is

3	6	3	6	0
4	0	6	0	2
6	4	3	6	0
2	3	8	3	0
0	2	0	0	2

(3 marks)

(b) Show that the zeros in the table in part (a) can be covered with three lines and use adjustments to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the two possible ways of allocating the five managers to the five centres with the least possible total travel cost. *(3 marks)*

(d) Find the value of this minimum daily total travel cost. *(1 mark)*

Turn over ▶

3 Two people, Rose and Callum, play a zero-sum game. The game is represented by the following pay-off matrix for Rose.

Callum

Rose		$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	5	2	-1
	$\mathbf{R_2}$	-3	-1	5
	$\mathbf{R_3}$	4	1	-2

(a) (i) State the play-safe strategy for Rose and give a reason for your answer. *(2 marks)*

(ii) Show that there is no stable solution for this game. *(2 marks)*

(b) Explain why Rose should never play strategy $\mathbf{R_3}$. *(1 mark)*

(c) Rose adopts a mixed strategy, choosing $\mathbf{R_1}$ with probability p and $\mathbf{R_2}$ with probability $1 - p$.

(i) Find expressions for the expected gain for Rose when Callum chooses each of his three possible strategies. Simplify your expressions. *(3 marks)*

(ii) Illustrate graphically these expected gains for $0 \leqslant p \leqslant 1$. *(2 marks)*

(iii) Hence determine the optimal mixed strategy for Rose. *(3 marks)*

(iv) Find the value of the game. *(1 mark)*

4 A linear programming problem involving variables x and y is to be solved. The objective function to be maximised is $P = 3x + 5y$. The initial Simplex tableau is given below.

P	***x***	***y***	***s***	***t***	***u***	***value***
1	−3	−5	0	0	0	0
0	1	2	1	0	0	36
0	1	1	0	1	0	20
0	4	1	0	0	1	39

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, write down **three** inequalities involving x and y for this problem. *(2 marks)*

(b) (i) By choosing the first pivot from the ***y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

Turn over for the next question

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A maker of exclusive furniture is planning to build three cabinets A, B and C at the rate of one per month. The order in which they are built is a matter of choice, but the costs will vary because of the materials available and suppliers' costs. The expected costs, in pounds, are given in the table.

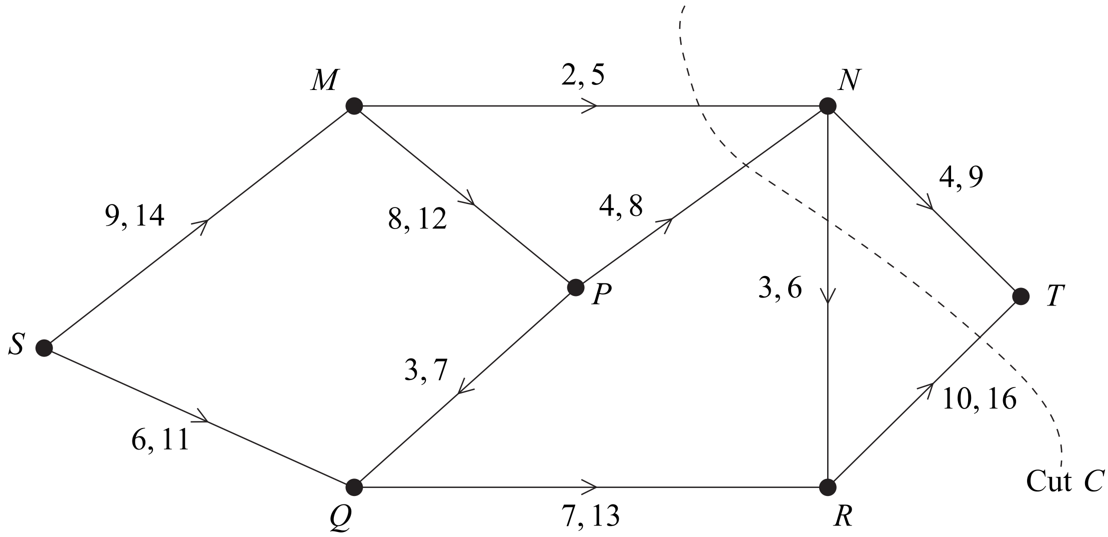
Month	**Already built**	**Cost**		
		A	***B***	***C***
1	–	500	440	475
2	***A*** ***B*** ***C***	– 510 520	440 – 490	490 500 –
3	***A* and *B*** ***A* and *C*** ***B* and *C***	– – 510	– 500 –	520 – –

(a) Use dynamic programming, working **backwards** from month 3, to determine the order of manufacture that **minimises** the total cost. You may wish to use **Figure 3** for your working. *(6 marks)*

(b) It is discovered that the figures given were actually the profits, not the costs, for each item. Modify your solution to find the order of manufacture that **maximises** the total profit. You may wish to use the final column of **Figure 3** for your working. *(4 marks)*

6 [**Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges MP, PN, QR and NR. *(4 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(2 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

ASSESSMENT and QUALIFICATIONS ALLIANCE

Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 1)

Activity	**Immediate Predecessors**
A	–
B	–
C	
D	
E	
F	
G	
H	
I	
J	
K	

Figure 2 (for use in Question 1)

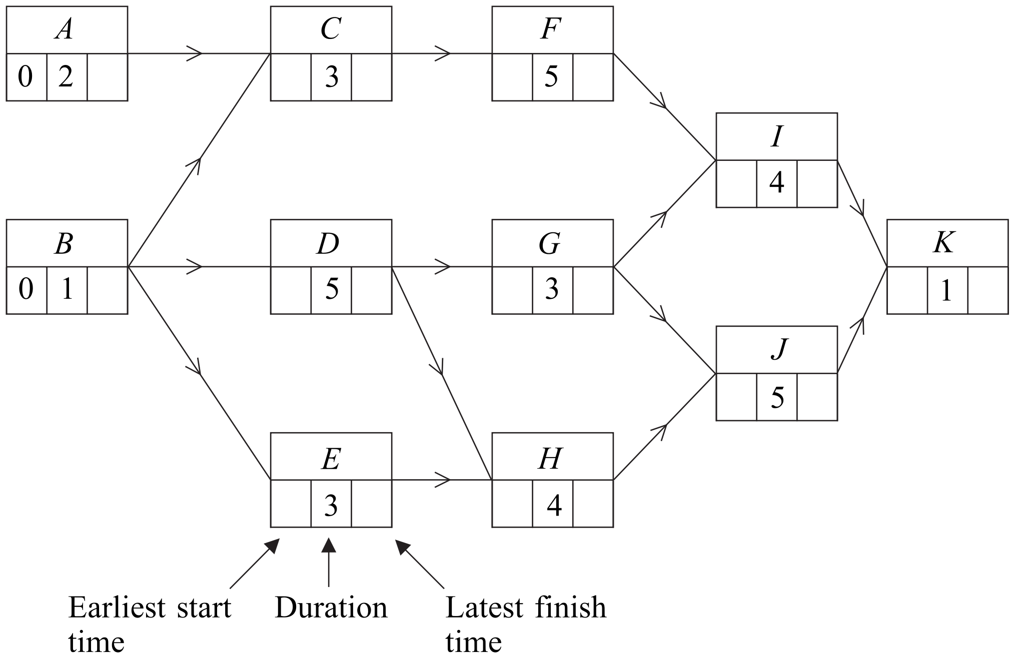


Figure 3 (for use in Question 5)

Month	**Already built**	**Machine built**	For use in part (a)		For use in part (b)
3	A and B	C			
	A and C	B			
	B and C	A			
2	A	B			
		C			
	B	A			
		C			
	C	A			
		B			

Turn over ▶

Figure 4 (for use in Question 6)

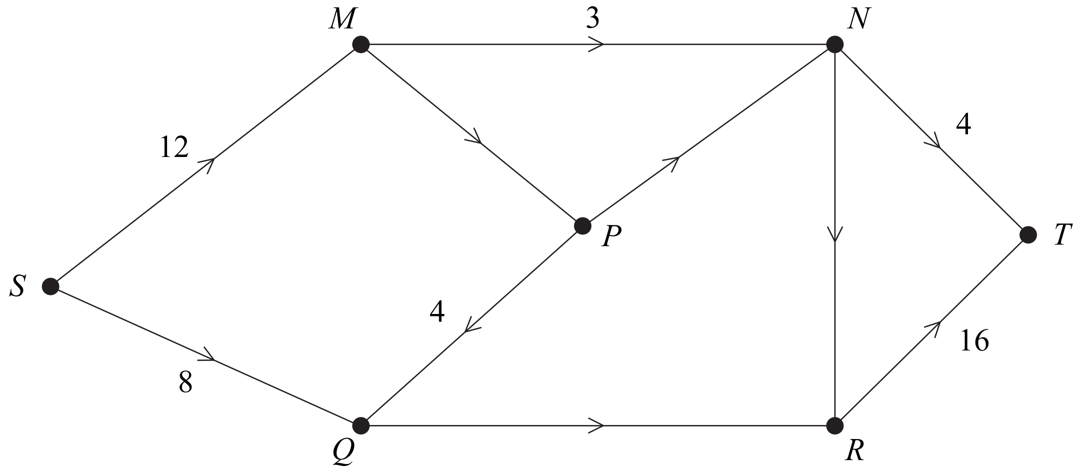
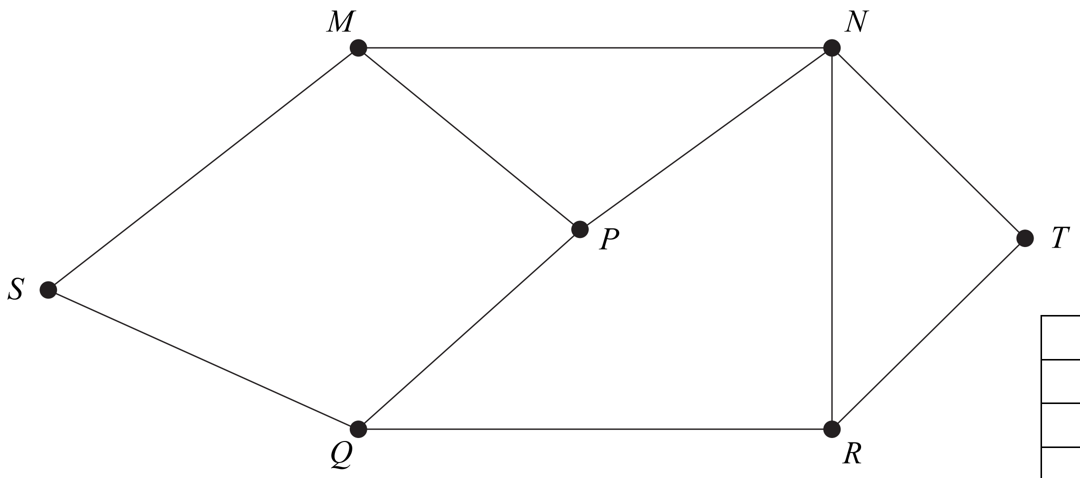
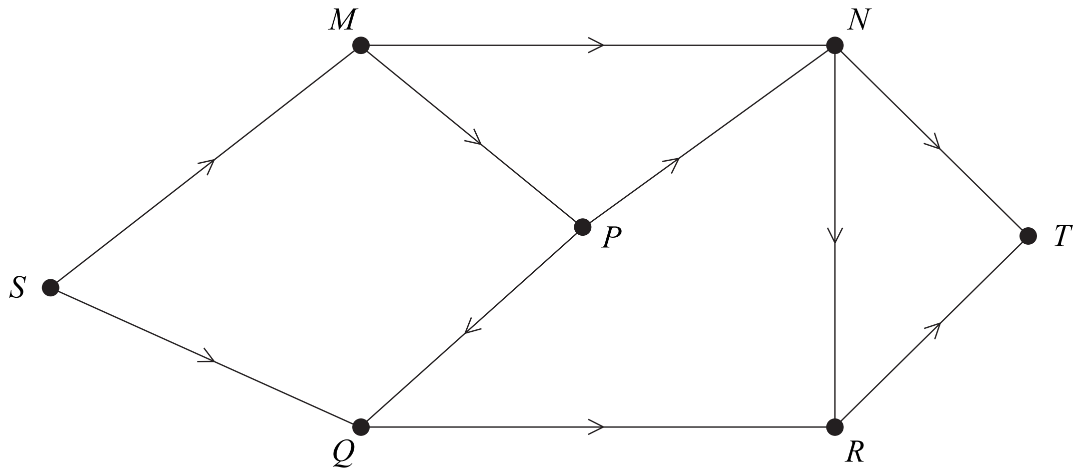


Figure 5 (for use in Question 6)



Path	Flow

Figure 6 (for use in Question 6)



General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Wednesday 30 January 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

A group of workers is involved in a building project. The table shows the activities involved. Each worker can perform any of the given activities.

Activity	**Immediate predecessors**	**Duration (days)**	**Number of workers required**
A	–	3	5
B	A	8	2
C	A	7	3
D	B, C	8	4
E	C	10	2
F	C	3	3
G	D, E	3	4
H	F	6	1
I	G, H	2	3

(a) Complete the activity network for the project on **Figure 1**. *(2 marks)*

(b) Find the earliest start time and the latest finish time for each activity, inserting their values on **Figure 1**. *(4 marks)*

(c) Find the critical path and state the minimum time for completion. *(2 marks)*

(d) The number of workers required for each activity is given in the table above. Given that each activity starts as early as possible and assuming there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2**, indicating clearly which activities take place at any given time. *(4 marks)*

(e) It is later discovered that there are only 7 workers available at any time. Use resource levelling to explain why the project will overrun and indicate which activities need to be delayed so that the project can be completed with the minimum extra time. State the minimum extra time required. *(3 marks)*

2 The following table shows the times taken, in minutes, by five people, Ash, Bob, Col, Dan and Emma, to carry out the tasks 1, 2, 3 and 4. Dan cannot do task 3.

	Ash	**Bob**	**Col**	**Dan**	**Emma**
Task 1	14	10	12	12	14
Task 2	11	13	10	12	12
Task 3	13	11	12	**	12
Task 4	13	10	12	13	15

Each of the four tasks is to be given to a different one of the five people so that the overall time for the four tasks is minimised.

(a) Modify the table of values by adding an extra row of **non-zero** values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) Use the Hungarian algorithm, reducing **columns first** then rows, to decide which four people should be allocated to which task. State the minimum total time for the four tasks using this matching. *(8 marks)*

(c) After special training, Dan is able to complete task 3 in 12 minutes. Determine, giving a reason, whether the minimum total time found in part (b) could be improved. *(2 marks)*

3 Two people, Rob and Con, play a zero-sum game.

The game is represented by the following pay-off matrix for Rob.

			Con	
	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	-2	5	3
Rob	$\mathbf{R_2}$	3	-3	-1
	$\mathbf{R_3}$	-3	3	2

(a) Explain what is meant by the term ‘zero-sum game’. *(1 mark)*

(b) Show that this game has no stable solution. *(3 marks)*

(c) Explain why Rob should never play strategy R_3. *(1 mark)*

(d) (i) Find the optimal mixed strategy for Rob. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

Turn over ▶

4 A linear programming problem involving the variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 3y + 5z$. The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−2	−3	−5	0	0	0	0
0	1	0	1	1	0	0	9
0	2	1	4	0	1	0	40
0	4	2	3	0	0	1	33

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot from the z-column, perform **one** iteration of the Simplex method. *(4 marks)*

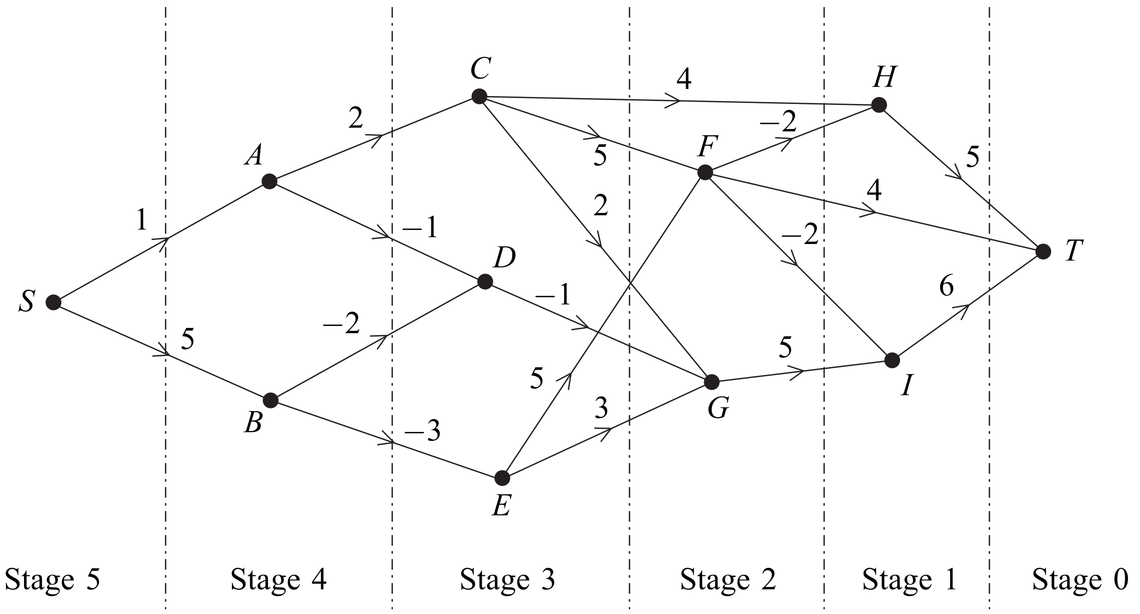
(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and state the values of the slack variables. *(3 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

The following network shows 11 vertices. The number on each edge is the cost of travelling between the corresponding vertices. A negative number indicates a reduction by the amount shown.



(a) **Working backwards from *T***, use dynamic programming to find the minimum cost of travelling from S to T. You may wish to complete the table on **Figure 3** as your solution. *(6 marks)*

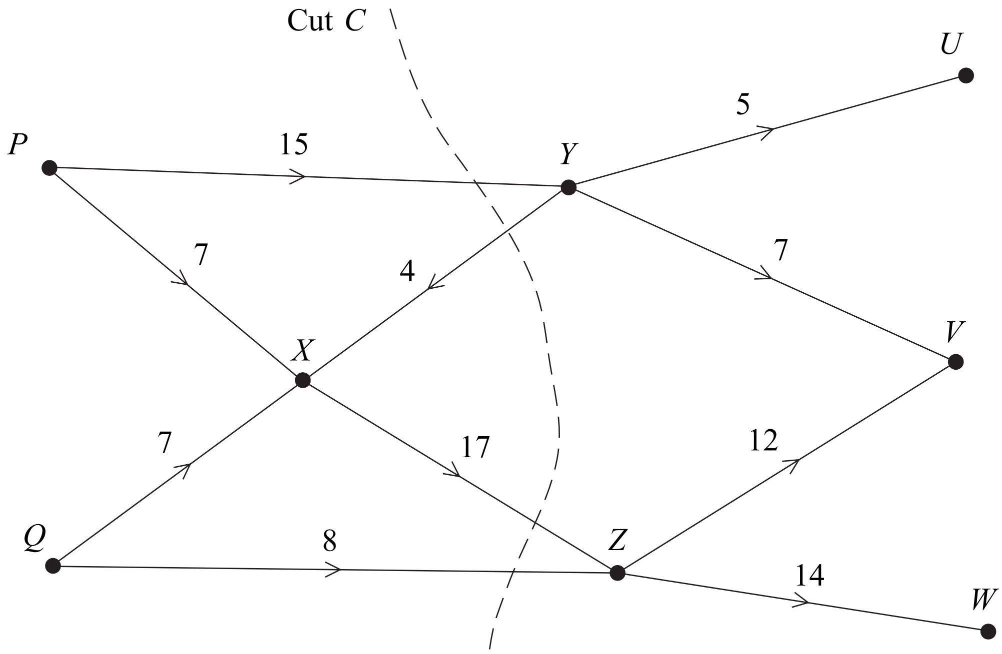
(b) State the minimum cost and the routes corresponding to this minimum cost. *(3 marks)*

Turn over for the next question

6 [**Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

Water has to be transferred from two mountain lakes P and Q to three urban reservoirs U, V and W. There are pumping stations at X, Y and Z.

The possible routes with the capacities along each edge, in millions of litres per hour, are shown in the following diagram.



(a) On **Figure 4**, add a super-source, S, and a super-sink, T, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each of the edges you have added. *(2 marks)*

(b) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(c) On **Figure 5**, write down the maximum flows along the routes $SQZWT$ and $SPYXZVT$. *(2 marks)*

(d) (i) On **Figure 6**, add the vertices S and T and the edges connecting S and T to the network. Using the maximum flows along the routes $SQZWT$ and $SPYXZVT$ found in part (c) as the initial flow, indicate the potential increases and decreases of flow on each edge. *(2 marks)*

(ii) Use flow augmentation to find the maximum flow from S to T. You should indicate any flow augmenting paths on **Figure 5** and modify the potential increases and decreases of the flow on **Figure 6**. *(4 marks)*

(e) State the value of the flow from Y to X in millions of litres per hour when the maximum flow is achieved. *(1 mark)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
January 2008
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

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Figure 1 (for use in Question 1)

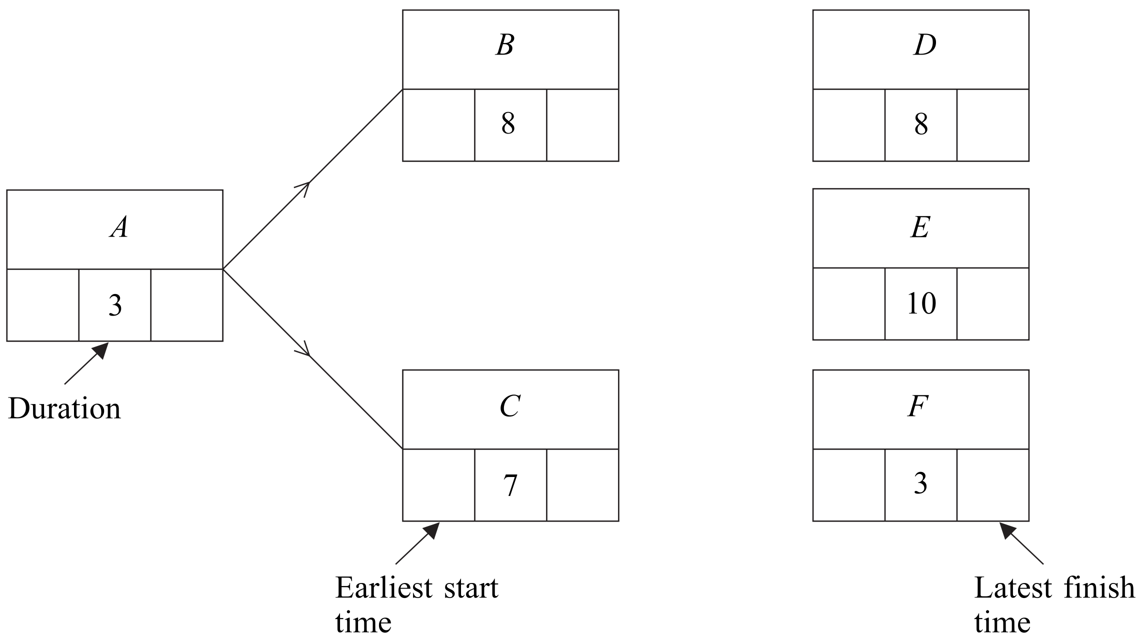


Figure 2 (for use in Question 1)

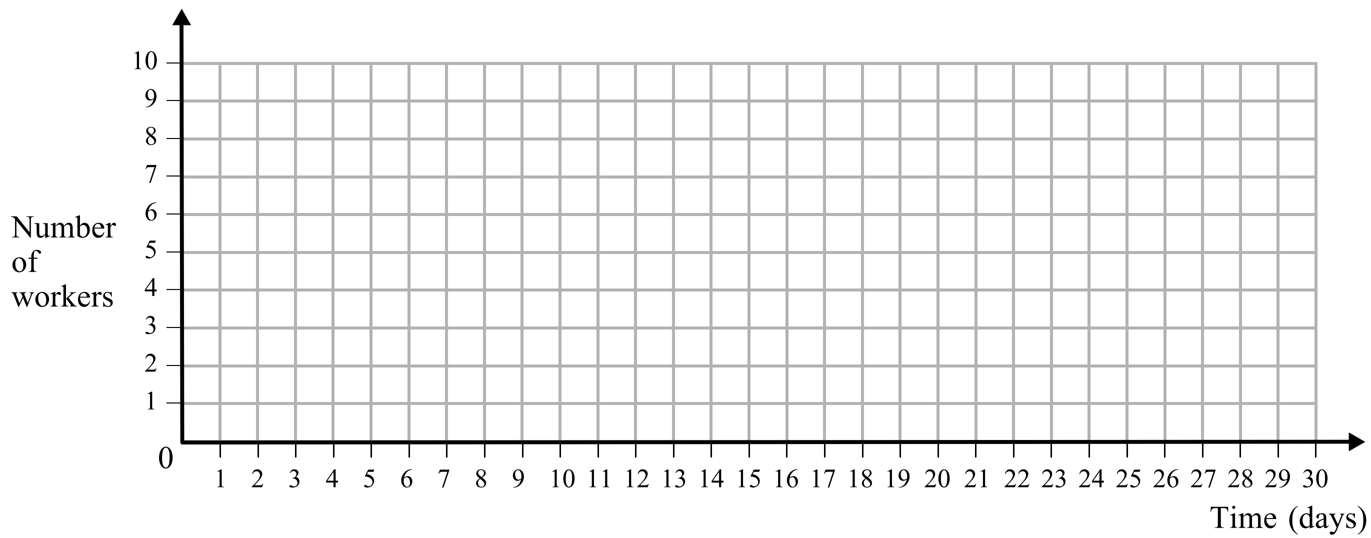


Figure 3 (for use in Question 5)

Stage	State	From	Value	
1	H	T	5	
	I	T	6	
2	F	H	$-2 + 5 = 3$	
		T	4	
		I		
	G	I		
3	C	H		
		F		
		G		

Turn over ▶

Figure 4 (for use in Question 6)

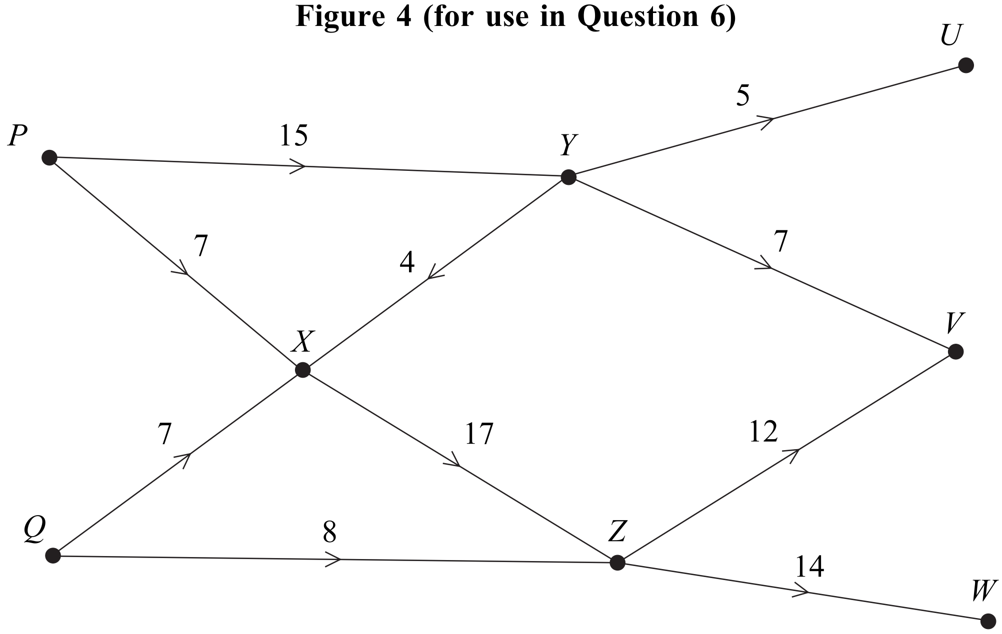
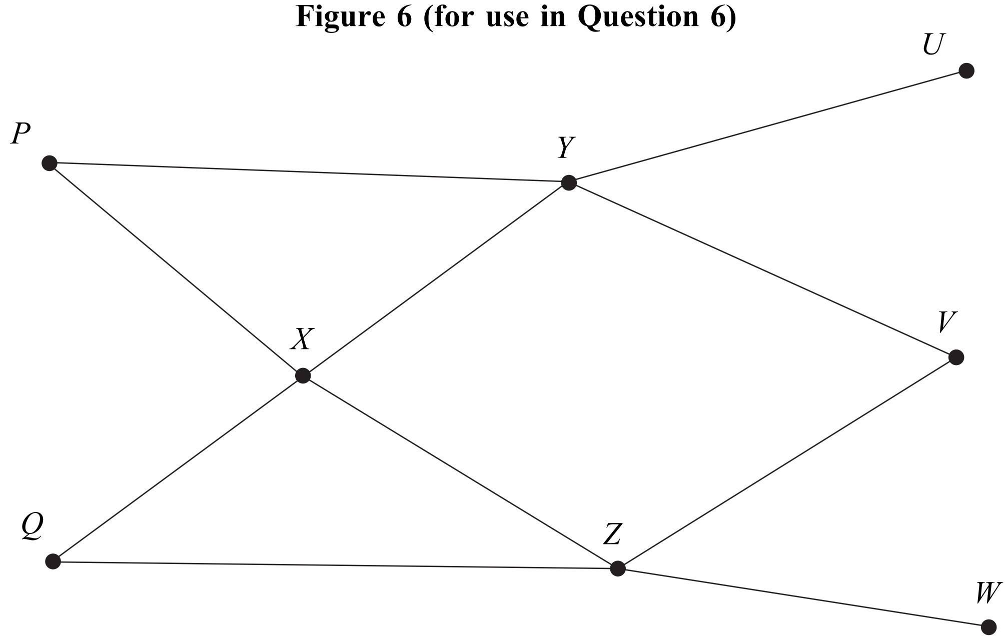


Figure 5 (for use in Question 6)

Route	**Flow**
SQZWT	
SPYXZVT	

Figure 6 (for use in Question 6)



General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Wednesday 21 May 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

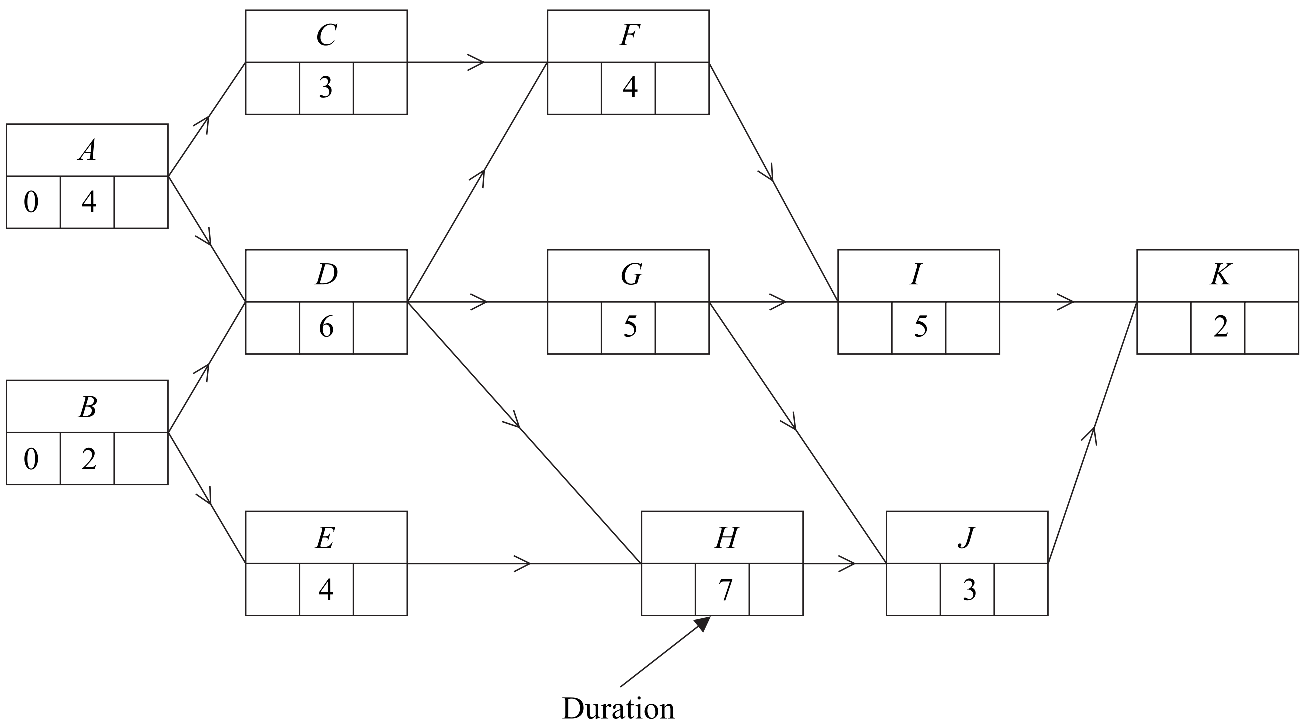
Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

The following diagram shows an activity network for a project. The time needed for each activity is given in days.



(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical paths and state the minimum time for completion. *(3 marks)*

(c) On **Figure 2**, draw a cascade diagram (Gantt chart) for the project, assuming each activity starts as early as possible. *(3 marks)*

(d) Activity C takes 5 days longer than first expected. Determine the effect on the earliest start time for other activities and the minimum completion time for the project. *(2 marks)*

2 The following table shows the scores of five people, Alice, Baji, Cath, Dip and Ede, after playing five different computer games.

	Alice	**Baji**	**Cath**	**Dip**	**Ede**
Game 1	17	16	19	17	20
Game 2	20	13	15	16	18
Game 3	16	17	15	18	13
Game 4	13	14	18	15	17
Game 5	15	16	20	16	15

Each of the five games is to be assigned to one of the five people so that the total score is maximised. No person can be assigned to more than one game.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $20 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 20, and hence show that, by reducing **columns first** and then rows, the resulting table of values is as below.

3	1	1	1	0
0	4	5	2	2
4	0	5	0	7
5	1	0	1	1
5	1	0	2	5

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with one horizontal and three vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of games to the five people so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

3 Two people, Roseanne and Collette, play a zero-sum game. The game is represented by the following pay-off matrix for Roseanne.

		Collette		
	Strategy	C_1	C_2	C_3
Roseanne	R_1	-3	2	3
	R_2	2	-1	-4

(a) (i) Find the optimal mixed strategy for Roseanne. *(7 marks)*

(ii) Show that the value of the game is -0.5. *(1 mark)*

(b) (i) Collette plays strategy C_1 with probability p and strategy C_2 with probability q. Write down, in terms of p and q, the probability that she plays strategy C_3. *(1 mark)*

(ii) Hence, given that the value of the game is -0.5, find the optimal mixed strategy for Collette. *(4 marks)*

4 A linear programming problem consists of maximising an objective function P involving three variables x, y and z. Slack variables s, t, u and v are introduced and the Simplex method is used to solve the problem. Several iterations of the method lead to the following tableau.

P	x	y	z	s	t	u	v	*value*
1	0	-12	0	5	-3	0	0	37
0	1	-8	0	1	2	0	0	16
0	0	4	0	0	3	0	1	20
0	0	2	0	-3	2	1	0	14
0	0	1	1	2	5	0	0	8

(a) (i) The pivot for the next iteration is chosen from the ***y*-column**. State which value should be chosen and explain the reason for your choice. *(2 marks)*

(ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) Explain why your new tableau solves the original problem. *(1 mark)*

(c) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(d) State the values of the slack variables at the optimum point. Hence determine how many of the original inequalities still have some slack when the optimum is reached. *(2 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A small firm produces high quality cabinets.

It can produce up to 4 cabinets each month.

Whenever at least one cabinet is made during that month, the overhead costs for that month are £300.

It is possible to hold in stock a maximum of 2 cabinets during any month.

The cost of storage is £50 per cabinet per month.

The orders for cabinets are shown in the table below. There is no stock at the beginning of January and the firm plans to clear all stock after completing the April orders.

Month	January	February	March	April
Number of cabinets required	3	3	5	2

(a) Determine the total cost of storing 2 cabinets and producing 3 cabinets in a given month. *(2 marks)*

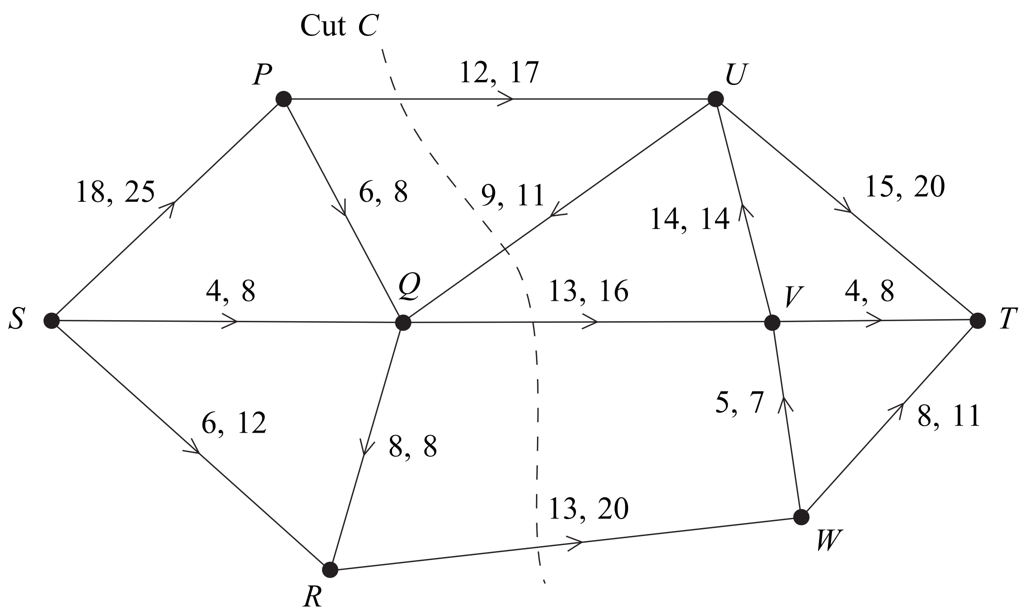
(b) By completing the table of values on **Figure 3**, or otherwise, use dynamic programming, **working backwards from April**, to find the production schedule which minimises total costs. *(8 marks)*

(c) Each cabinet is sold for £2000 but there is an additional cost of £300 for materials to make each cabinet and £2000 per month in wages. Determine the total profit for the four-month period. *(3 marks)*

Turn over for the next question

6 [**Figures 4**, **5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) Hence state what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 32 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges PQ, UQ and UT. *(3 marks)*

(c) (i) Taking your feasible flow from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(1 mark)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS
Unit Decision 2

MD02

ASSESSMENT and QUALIFICATIONS ALLIANCE

Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 1)

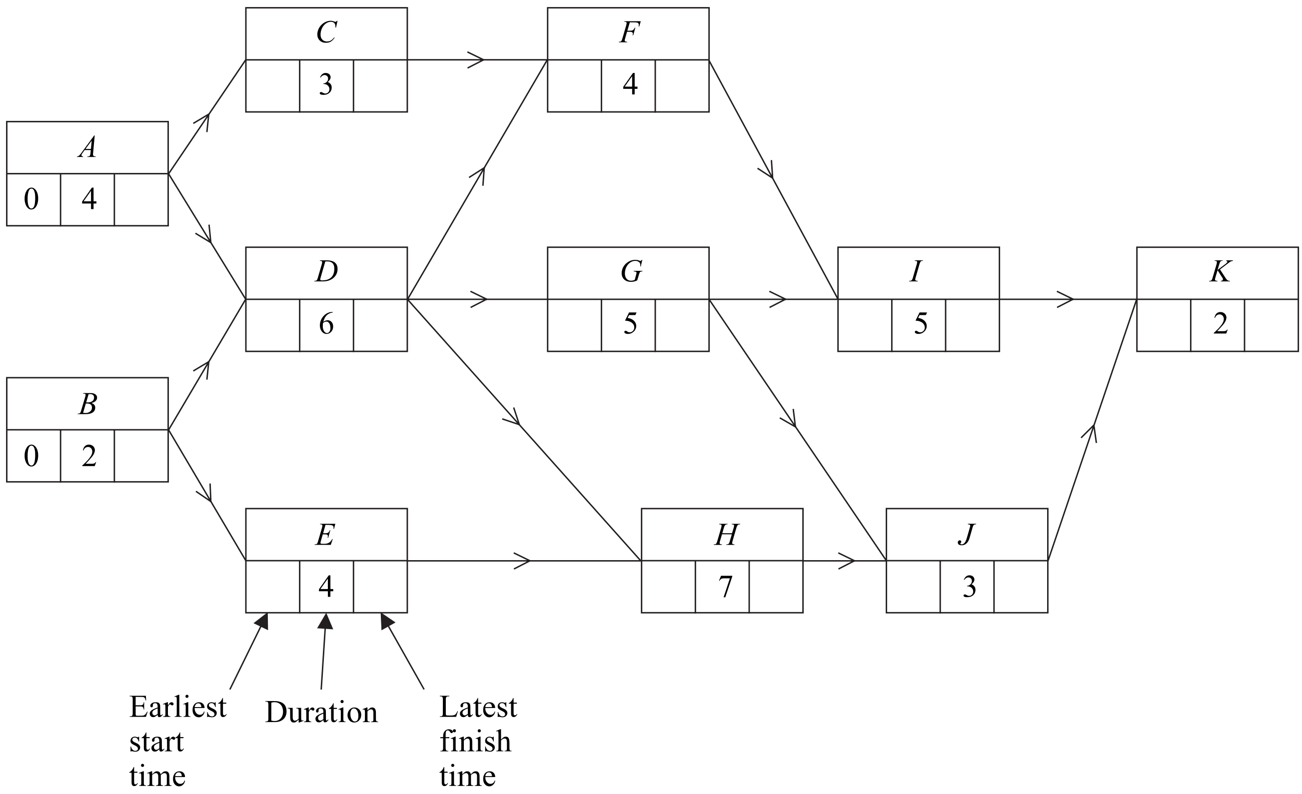


Figure 2 (for use in Question 1)

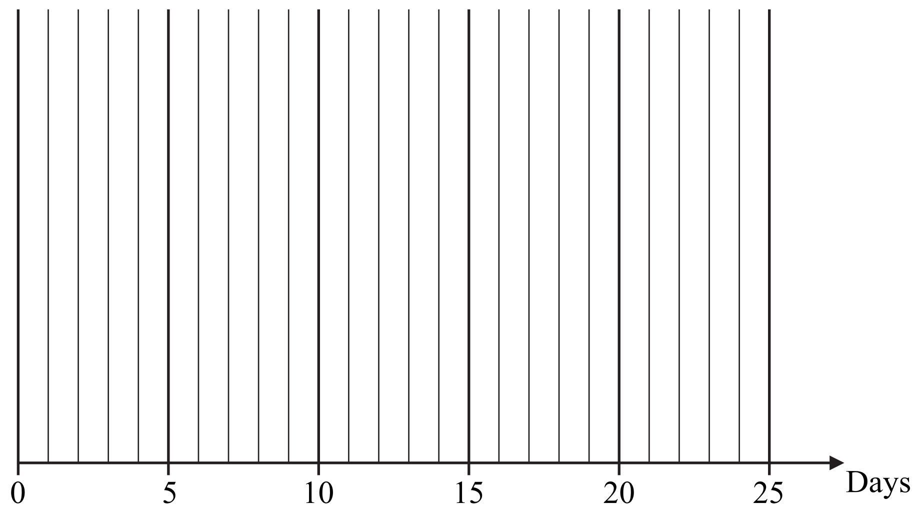


Figure 3 (for use in Question 5)

Initial State is the number of cabinets in stock at the beginning of that month.

Action is the number of cabinets made during that month.

Destination State is the number of cabinets in stock at the end of that month after the demand has been met.

The destination state for March becomes the initial state for April and so on.

Month & Demand	Initial State	Action	Destination State	Value
April	0	2	0	$300 + 0 = 300$
(demand 2)				
	1	1	0	$300 + 50 = 350$
	2	0	0	$0 + 100 = 100$
March	1	4	0	$300 + 50 + 300 = 650$
(demand 5)				
	2	3	0	
		4	1	
February	0	4	1	
(demand 3)				
	1	3	1	
		4	2	
	2	2	1	
		3	2	
January	0	3	0	
(demand 3)		4	1	

Production Schedule which minimises total costs

Month	January	February	March	April
Number of cabinets made				

Turn over ▶

Figure 4 (for use in Question 6)

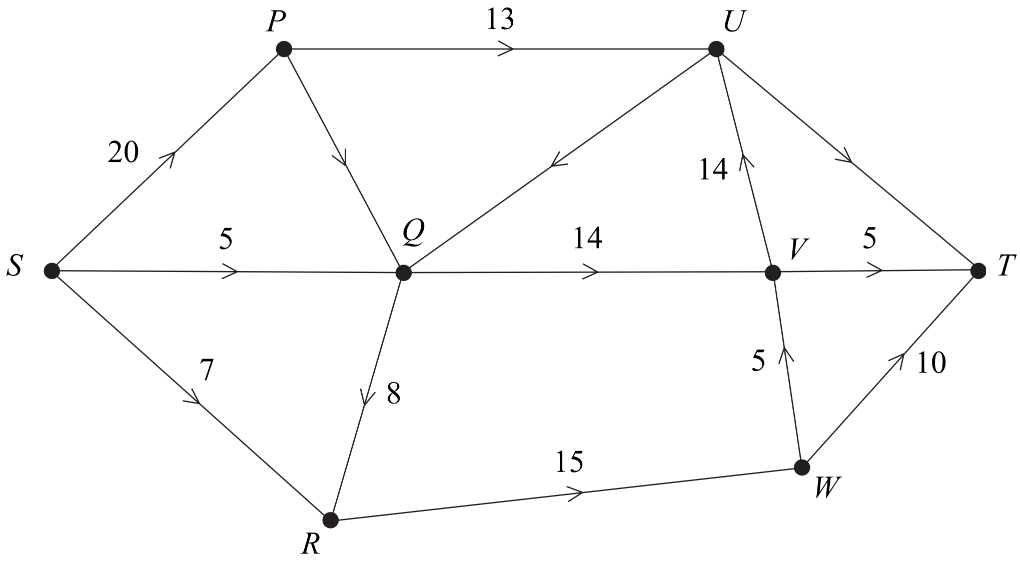
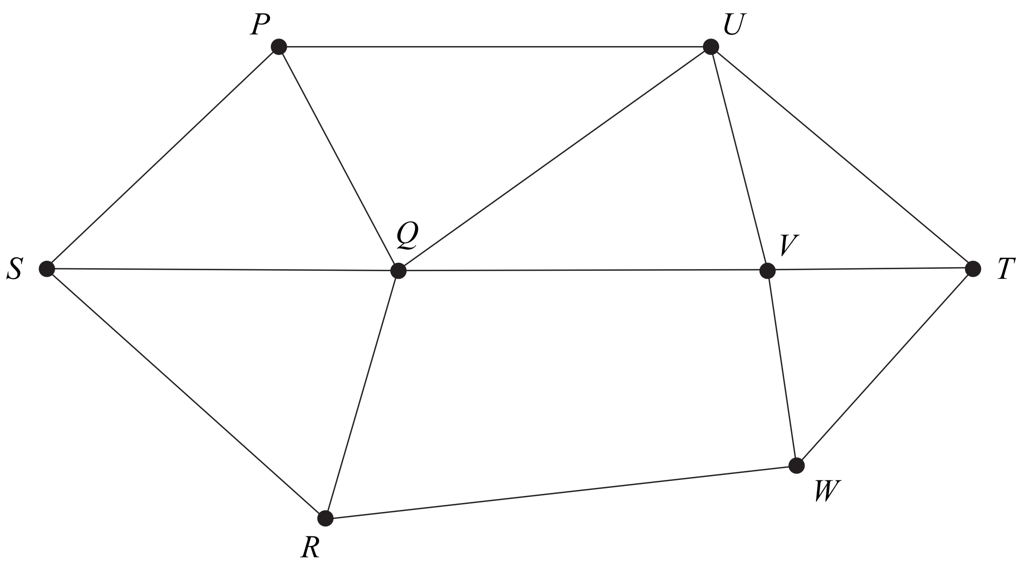
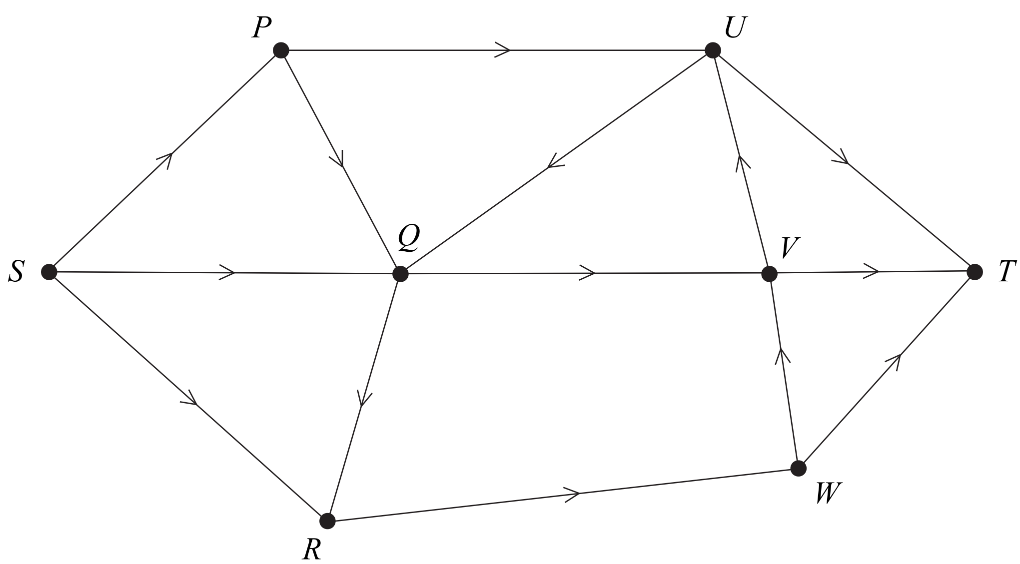


Figure 5 (for use in Question 6)



Path	Additional Flow

Figure 6 (for use in Question 6)



General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Thursday 29 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 2, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 The times taken in minutes for five people, P, Q, R, S and T, to complete each of five different tasks are recorded in the table.

	P	**Q**	**R**	**S**	**T**
Task 1	17	20	19	17	17
Task 2	19	18	18	18	15
Task 3	13	16	16	14	12
Task 4	13	13	15	13	13
Task 5	10	11	12	14	13

Using the Hungarian algorithm, each of the five people is to be allocated to a different task so that the total time for completing the five tasks is minimised.

(a) By reducing the **columns first** and then the rows, show that the new table of values is as follows.

3	5	3	0	1
6	4	3	2	0
3	5	4	1	0
3	2	3	0	1
0	0	0	1	1

(3 marks)

(b) Show that the zeros in the table in part (a) can be covered with three lines, and use adjustments to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the two possible ways of allocating the five people to the five tasks so that the total completion time is minimised. *(3 marks)*

(d) Find the minimum total time for completing the five tasks. *(1 mark)*

2 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

Figure 1 shows the activity network and the duration in days of each activity for a particular project.

(a) On **Figure 1**:

(i) find the earliest start time for each activity; *(2 marks)*

(ii) find the latest finish time for each activity. *(2 marks)*

(b) Find the critical paths and state the minimum time for completion. *(3 marks)*

(c) The number of workers required for each activity is shown in the table.

Activity	A	B	C	D	E	F	G	H	I	J
Number of workers required	3	3	4	2	3	4	1	2	2	5

(i) Given that each activity starts as early as possible and assuming that there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2**, indicating clearly which activities take place at any given time. *(4 marks)*

(ii) It is later discovered that there are only 6 workers available at any time. Explain why the project will overrun, and use resource levelling to indicate which activities need to be delayed so that the project can be completed with the minimum extra time. State the minimum extra time required. *(3 marks)*

Turn over for the next question

3 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 4x - 5y + 6z$

subject to $6x + 7y - 4z \leqslant 30$

$2x + 4y - 5z \leqslant 8$

$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$ *(3 marks)*

(b) The Simplex method is to be used to solve this problem.

(i) Explain why it is not possible to choose a pivot from the z-column initially. *(1 mark)*

(ii) Identify the initial pivot and explain why this particular element should be chosen. *(2 marks)*

(iii) Perform one iteration using your initial tableau from part (a). *(3 marks)*

(iv) State the values of x, y and z after this first iteration. *(2 marks)*

(v) Without performing any further iterations, explain why P has no finite maximum value. *(1 mark)*

(c) Using the same inequalities as in part (a), the problem is modified to

Maximise $Q = 4x - 5y - 20z$

(i) Write down a modified initial tableau and the revised tableau after one iteration. *(2 marks)*

(ii) Hence find the maximum value of Q. *(1 mark)*

4 (a) Two people, Raj and Cal, play a zero-sum game. The game is represented by the following pay-off matrix for Raj.

Cal

	Strategy	**X**	**Y**	**Z**
	I	−7	8	−5
Raj	**II**	6	2	−1
	III	−2	4	−3

Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) Ros and Carly play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Ros, where x is a constant.

Carly

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$
Ros	$\mathbf{R_1}$	5	x
	$\mathbf{R_2}$	−2	4

Ros chooses strategy R_1 with probability p.

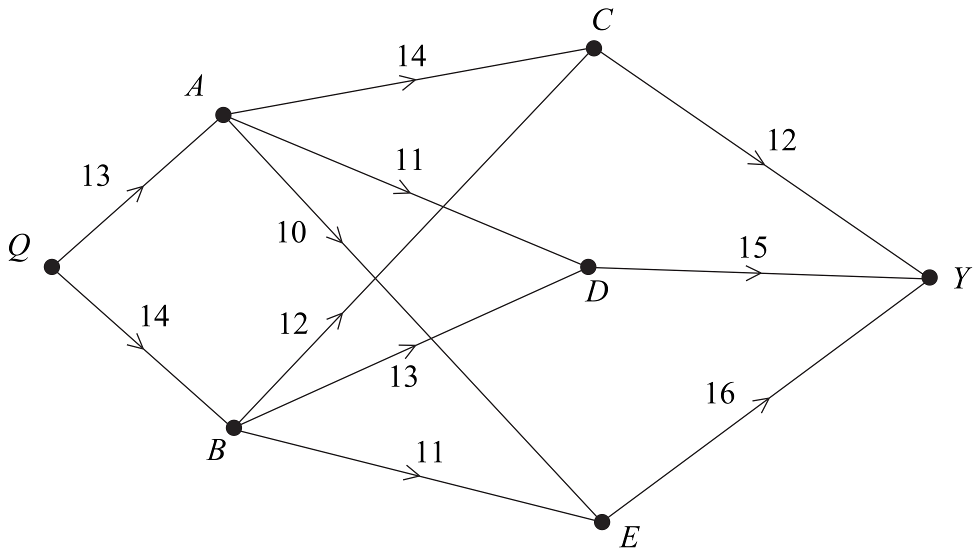
(i) Find expressions for the expected gains for Ros when Carly chooses each of the strategies C_1 and C_2. *(2 marks)*

(ii) Given that the value of the game is $\frac{8}{3}$, find the value of p and the value of x. *(4 marks)*

Turn over for the next question

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

A truck has to transport stones from a quarry, Q, to a builders yard, Y. The network shows the possible roads from Q to Y. Along each road there are bridges with weight restrictions. The value on each edge indicates the maximum load in tonnes that can be carried by the truck along that particular road.



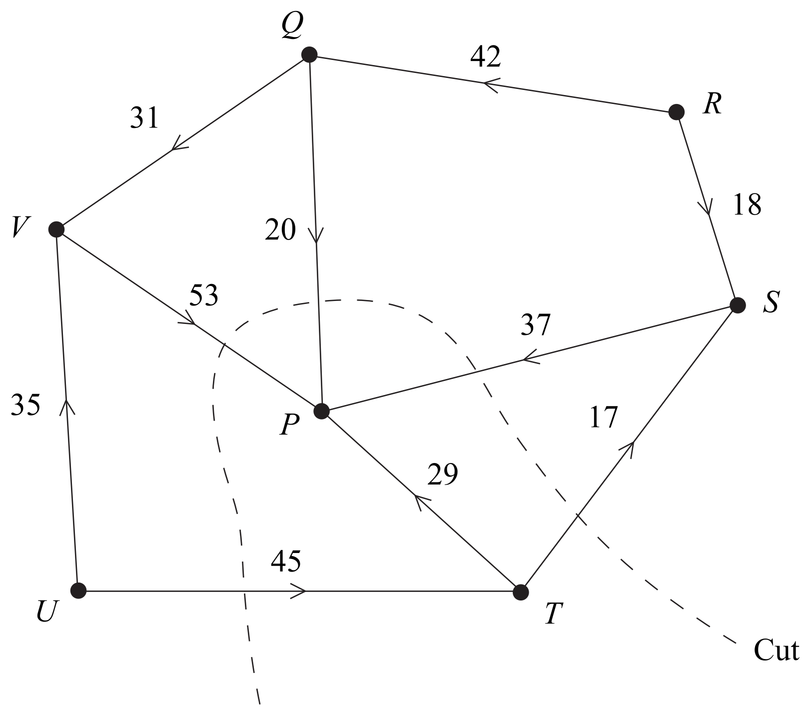
The truck is able to carry a load of up to 20 tonnes. The optimal route, known as the maximin route, is that for which the possible load that the truck can carry is as large as possible.

(a) Explain why the route $QACY$ is better than the route $QBEY$. *(2 marks)*

(b) By completing the table on **Figure 3**, or otherwise, use dynamic programming, **working backwards from *Y***, to find the optimal (maximin) route from Q to Y. Write down the maximin route and state the maximum possible load that the truck can carry from Q to Y. *(8 marks)*

6 [**Figures 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows the routes along corridors from two arrival gates to the passport control area, P, in a small airport. The number on each edge represents the maximum number of passengers that can travel along a particular corridor in one minute.



(a) State the vertices that represent the arrival gates. *(1 mark)*

(b) Find the value of the cut shown on the network. *(1 mark)*

(c) State the maximum flow along each of the routes $UTSP$ and $RQVP$. *(2 marks)*

(d) (i) Taking your answers to part (c) as the initial flow, use the labelling procedure on **Figure 4** to find the maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(6 marks)*

(ii) State the value of the maximum flow, and, on **Figure 5**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(e) On a particular day, there is an obstruction allowing no more than 50 passengers per minute to pass through vertex V. State the maximum number of passengers that can move through the network per minute on this particular day. *(2 marks)*

END OF QUESTIONS

General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MD02**
Unit Decision 2

Monday 15 June 2009 1.30 pm to 3.00 pm

For this paper you must have: • an 8-page answer book • the blue AQA booklet of formulae and statistical tables • an insert for use in Questions 1, 5 and 6 (enclosed). You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Answer **all** questions.

1 [**Figure 1**, printed on the insert, is provided for use in this question.]

A decorating project is to be undertaken. The table shows the activities involved.

Activity	**Immediate Predecessors**	**Duration (days)**
A	–	5
B	–	3
C	–	2
D	A, B	4
E	B, C	1
F	D	2
G	E	9
H	F, G	1
I	H	6
J	H	5
K	I, J	2

(a) Complete an activity network for the project on **Figure 1**. *(3 marks)*

(b) On **Figure 1**, indicate:

(i) the earliest start time for each activity; *(2 marks)*

(ii) the latest finish time for each activity. *(2 marks)*

(c) State the minimum completion time for the decorating project and identify the critical path. *(2 marks)*

(d) Activity F takes 4 days longer than first expected.

(i) Determine the new earliest start time for activities H and I. *(2 marks)*

(ii) State the minimum delay in completing the project. *(1 mark)*

2 Two people, Rowena and Colin, play a zero-sum game.

The game is represented by the following pay-off matrix for Rowena.

Colin

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	-4	5	4
Rowena	$\mathbf{R_2}$	2	-3	-1
	$\mathbf{R_3}$	-5	4	3

(a) Explain what is meant by the term 'zero-sum game'. *(1 mark)*

(b) Determine the play-safe strategy for Colin, giving a reason for your answer. *(2 marks)*

(c) Explain why Rowena should never play strategy R_3. *(1 mark)*

(d) Find the optimal mixed strategy for Rowena. *(7 marks)*

Turn over for the next question

3 Five lecturers were given the following scores when matched against criteria for teaching five courses in a college.

	Course 1	**Course 2**	**Course 3**	**Course 4**	**Course 5**
Ron	13	13	9	10	13
Sam	13	14	12	17	15
Tom	16	10	8	14	14
Una	11	14	12	16	10
Viv	12	14	14	13	15

Each lecturer is to be allocated to exactly one of the courses so as to maximise the total score of the five lecturers.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $17 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 17. Hence show that, by reducing **rows first** and then columns, the resulting table of values is as below.

0	0	3	3	0
4	3	4	0	2
0	6	7	2	2
5	2	3	0	6
3	1	0	2	0

(3 marks)

(c) Show that the zeros in the table in part (b) can be covered with two horizontal and two vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) Hence find the possible allocations of courses to the five lecturers so that the total score is maximised. *(4 marks)*

(e) State the value of the maximum total score. *(1 mark)*

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 4x + y + kz$, where k is a constant. The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***value***
1	-4	-1	$-k$	0	0	0
0	1	2	3	1	0	7
0	2	1	4	0	1	10

(a) In addition to $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$, write down **two** inequalities involving x, y and z for this problem. *(1 mark)*

(b) (i) The first pivot is chosen from the ***x*-column**. Identify the pivot and perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value of P has not been reached after this first iteration, find the possible values of k. *(2 marks)*

(c) Given that $k = 10$:

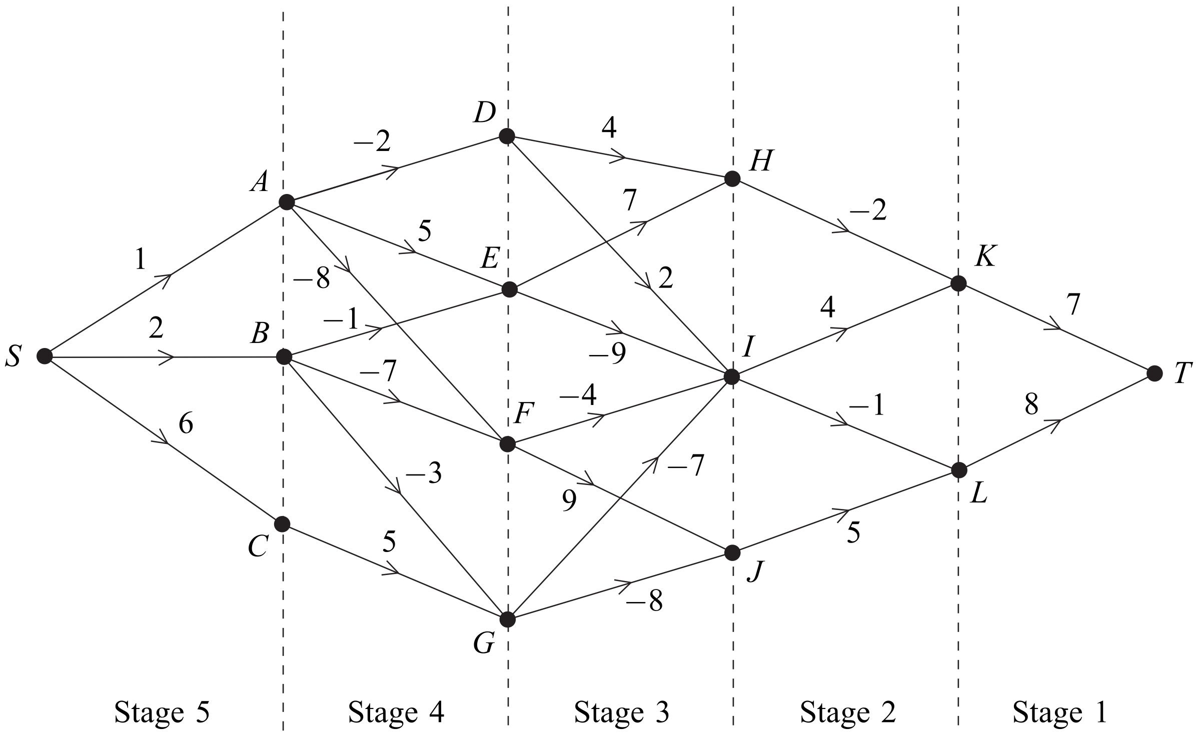
(i) perform one further iteration of the Simplex method; *(4 marks)*

(ii) interpret the final tableau. *(3 marks)*

Turn over for the next question

5 [**Figure 2**, printed on the insert, is provided for use in this question.]

A company has a number of stores. The following network shows the possible actions and profits over the next five years. The number on each edge is the expected profit, in millions of pounds. A negative number indicates a loss due to investment in new stores.

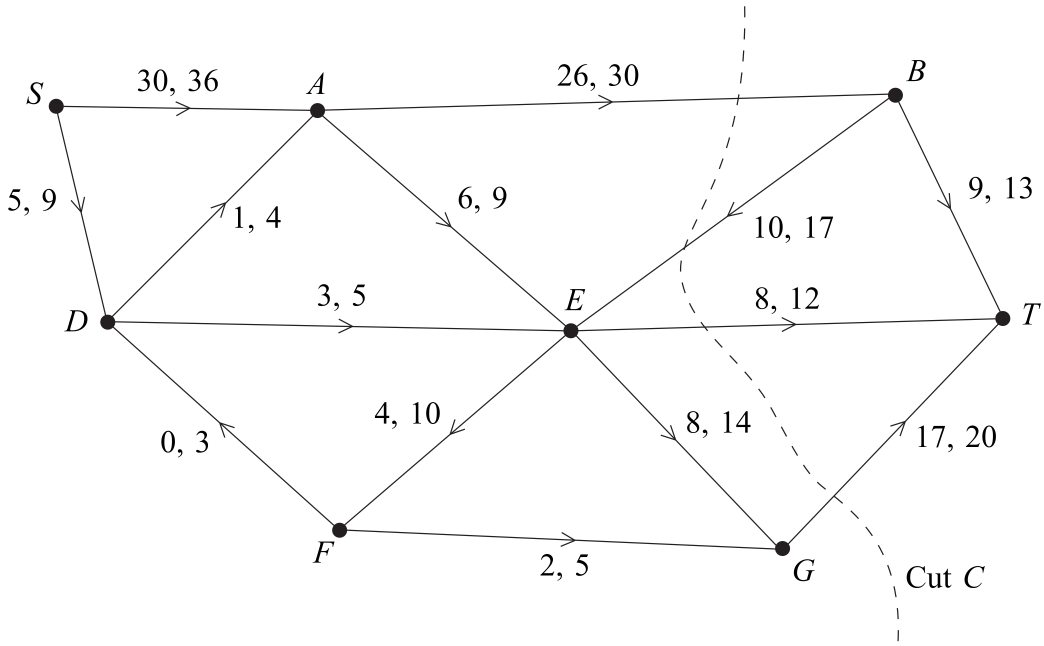


(a) **Working backwards from *T***, use dynamic programming to maximise the expected profits over the five years. You may wish to complete the table on **Figure 2** as your solution. *(7 marks)*

(b) State the maximum expected profit and the sequence of vertices from S to T in order to achieve this. *(2 marks)*

6 [**Figures 3, 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut C. *(2 marks)*

(b) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 40 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges AE, EF and FG. *(3 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4**. *(3 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

(d) Illustrate the maximum flow on **Figure 5**. *(2 marks)*

(e) Find a cut with value equal to that of the maximum flow. *(2 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS
Unit Decision 2

MD02

Insert

Insert for use in **Questions 1, 5 and 6**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 1)

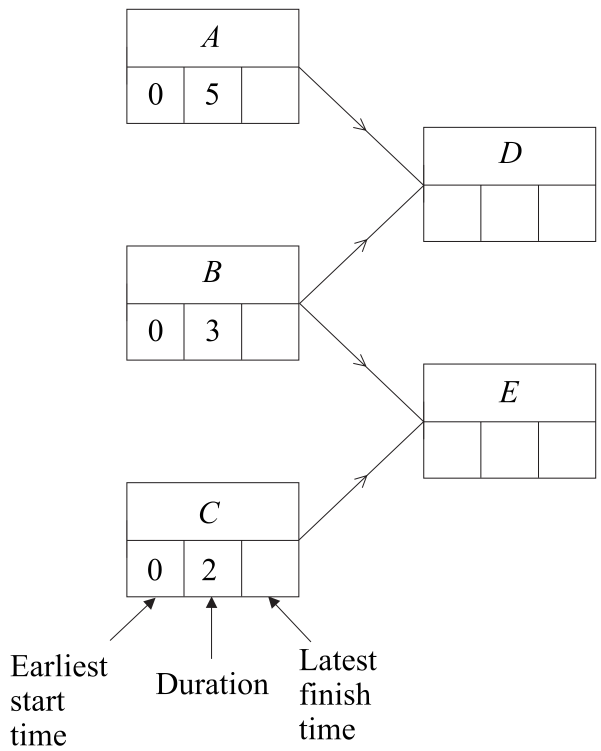


Figure 2 (for use in Question 5)

Stage	State	From	Value	
1	K	T	7	
	L	T	8	
2	H	K	$-2+7=5$	
	I	K		
		L		
	J	L		
3	D	H		
		I		

Turn over ▶

Figure 3 (for use in Question 6)

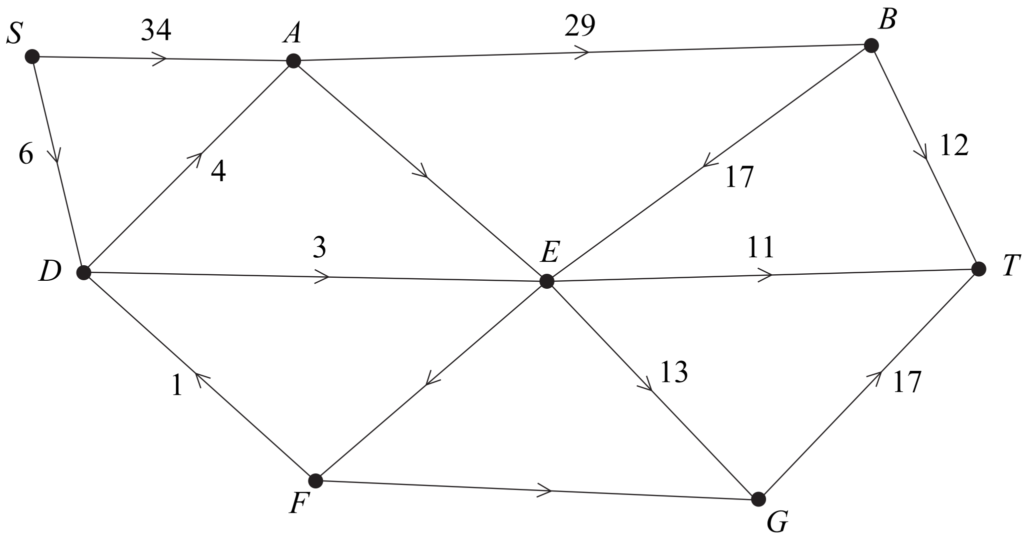
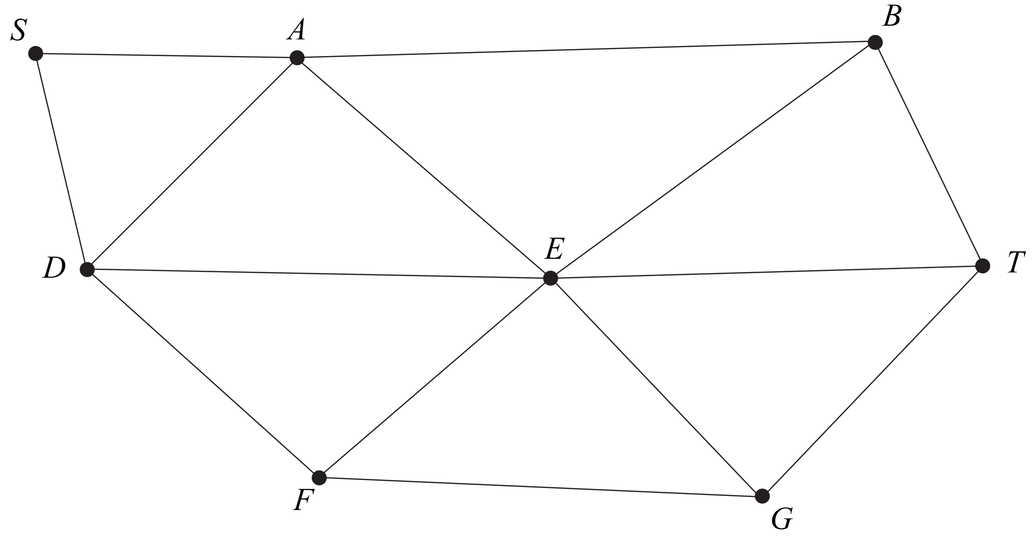
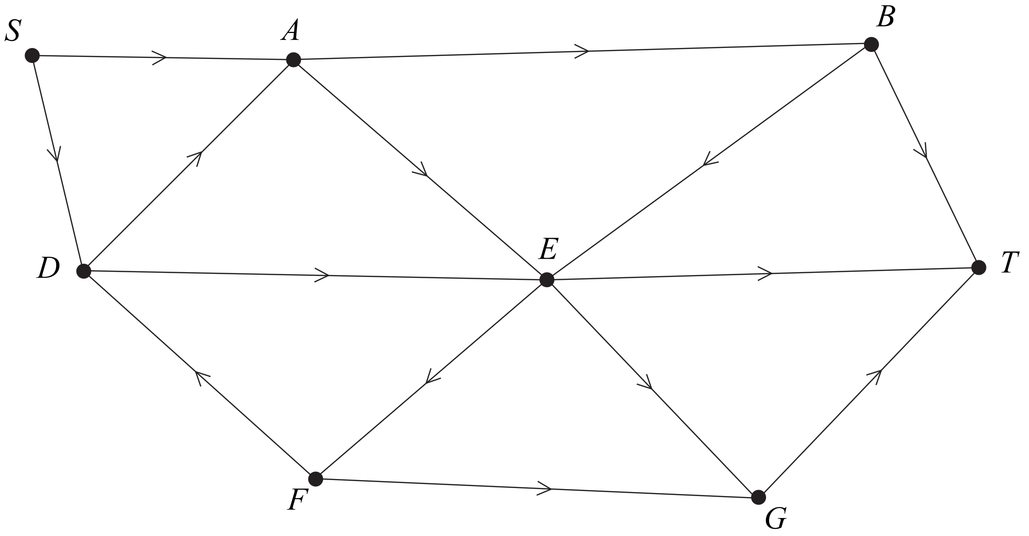


Figure 4 (for use in Question 6)



Path	Extra Flow

Figure 5 (for use in Question 6)



General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MD02

Unit Decision 2

Wednesday 20 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 1, 5 and 6 (enclosed).

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MD02.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Fill in the boxes at the top of the insert.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions.

1 [**Figures 1 and 2**, printed on the insert, are provided for use in this question.]

Figure 1 shows the activity network and the duration, in days, of each activity for a particular project.

(a) On **Figure 1**:

(i) find the earliest start time for each activity; *(2 marks)*

(ii) find the latest finish time for each activity. *(2 marks)*

(b) Find the float for activity G. *(1 mark)*

(c) Find the critical paths and state the minimum time for completion. *(3 marks)*

(d) The number of workers required for each activity is shown in the table.

Activity	A	B	C	D	E	F	G	H	I	J
Number of workers required	2	2	3	2	3	2	1	3	5	2

Given that each activity starts as **late** as possible and assuming that there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2**, indicating clearly which activities take place at any given time. *(5 marks)*

2 The following table shows the times taken, in minutes, by five people, Ron, Sam, Tim, Vic and Zac, to carry out the tasks 1, 2, 3 and 4. Sam takes x minutes, where $8 \leqslant x \leqslant 12$, to do task 2.

	Ron	**Sam**	**Tim**	**Vic**	**Zac**
Task 1	8	7	9	10	8
Task 2	9	x	8	7	11
Task 3	12	10	9	9	10
Task 4	11	9	8	11	11

Each of the four tasks is to be given to a different one of the five people so that the total time for the four tasks is minimised.

(a) Modify the table of values by adding an extra row of **non-zero** values so that the Hungarian algorithm can be applied. *(1 mark)*

(b) (i) Use the Hungarian algorithm, reducing **columns first** and then rows, to reduce the matrix to a form, in terms of x, from which the optimum matching can be made. *(5 marks)*

(ii) Hence find the possible way of allocating the four tasks so that the total time is minimised. *(2 marks)*

(iii) Find the minimum total time. *(1 mark)*

(c) After special training, Sam is able to complete task 2 in 7 minutes and is assigned to task 2.

Determine the possible ways of allocating the other three tasks so that the total time is minimised. *(2 marks)*

Turn over for the next question

3 (a) Two people, Ann and Bill, play a zero-sum game. The game is represented by the following pay-off matrix for Ann.

Bill

	Strategy	**B_1**	**B_2**	**B_3**
	A_1	-1	0	-2
Ann	**A_2**	4	-2	-3
	A_3	-4	-5	-3

Show that this game has a stable solution and state the play-safe strategies for Ann and Bill. *(4 marks)*

(b) Russ and Carlos play a different zero-sum game, which does not have a stable solution. The game is represented by the following pay-off matrix for Russ.

Carlos

	Strategy	**C_1**	**C_2**	**C_3**
Russ	**R_1**	-4	7	-3
	R_2	2	-1	1

(i) Find the optimal mixed strategy for Russ. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 4y + 3z$. The initial Simplex tableau is given below.

P	x	y	z	s	t	u	*value*
1	−2	−4	−3	0	0	0	0
0	2	2	1	1	0	0	14
0	−1	1	2	0	1	0	6
0	4	4	3	0	0	1	29

(a) (i) What name is given to the variables s, t and u? *(1 mark)*

(ii) Write down an equation involving x, y, z and s for this problem. *(1 mark)*

(b) (i) By choosing the first pivot **from the y-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Explain how you know that the optimal value has not been reached. *(1 mark)*

(c) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau, stating the values of P, x, y and z. *(3 marks)*

5 [**Figure 3**, printed on the insert, is provided for use in this question.]

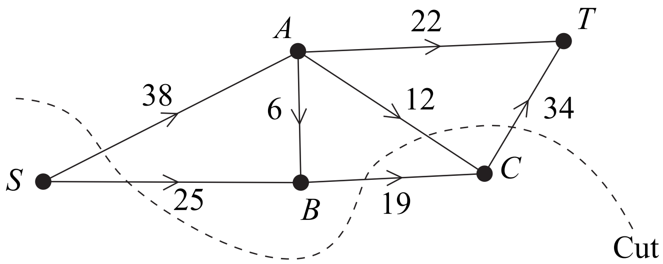
A landscape gardener has three projects, A, B and C, to be completed over a period of 4 months: May, June, July and August. The gardener must allocate one of these months to each project and the other month is to be taken as a holiday. Various factors, such as availability of materials and transport, mean that the costs for completing the projects in different months will vary. The costs, in thousands of pounds, are given in the table.

	May	**June**	**July**	**August**
Project *A*	17	16	18	16
Project *B*	14	13	12	10
Project *C*	14	17	15	14

By completing the table of values on **Figure 3**, or otherwise, use dynamic programming, **working backwards from August**, to find the project schedule that minimises total costs. State clearly which month should be taken as a holiday and which project should be undertaken in which month. *(10 marks)*

6 [**Figures 4, 5, 6 and 7**, printed on the insert, are provided for use in this question.]

(a) The network shows a flow from S to T along a system of pipes, with the capacity, in litres per minute, indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 97. *(1 mark)*

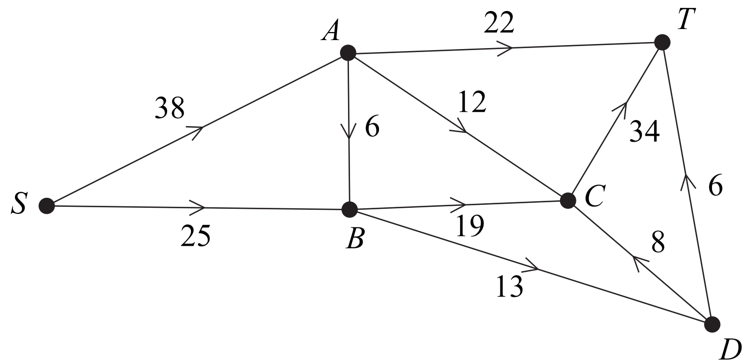
(ii) The cut shown on the diagram can be represented as $\{S, C\}$, $\{A, B, T\}$.

Complete the table on **Figure 4**, giving the value of each of the 8 possible cuts. *(4 marks)*

(iii) State the value of the maximum flow through the network, giving a reason for your answer. *(2 marks)*

(iv) Indicate on **Figure 5** a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(b) Extra pipes, BD, CD and DT, are added to form a new system, with the capacity, in litres per minute, indicated on each edge of the network below.



(i) Taking your values from **Figure 5** as the initial flow, use the labelling procedure on **Figure 6** to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

(ii) State the value of the new maximum flow, and, on **Figure 7**, indicate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MD02

Unit Decision 2

Friday 18 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Answer **all** questions in the spaces provided.

1 **Figure 1** below shows an activity diagram for a construction project. The time needed for each activity is given in days.

(a) Find the earliest start time and latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical paths and state the minimum time for completion of the project. *(3 marks)*

(c) On **Figure 2** opposite, draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as early as possible. *(3 marks)*

(d) A delay in supplies means that Activity I takes 9 days instead of 2.

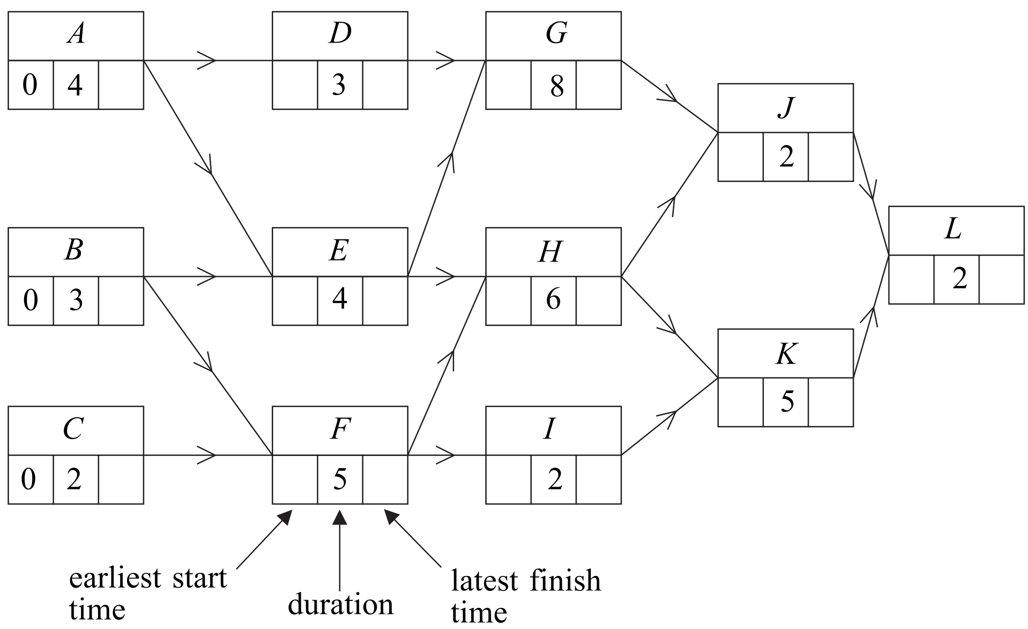
(i) Determine the effect on the **earliest** possible starting times for activities K and L. *(2 marks)*

(ii) State the number of days by which the completion of the project is now delayed. *(1 mark)*

QUESTION PART REFERENCE

(a)

Figure 1



(b) Critical paths are ....

....

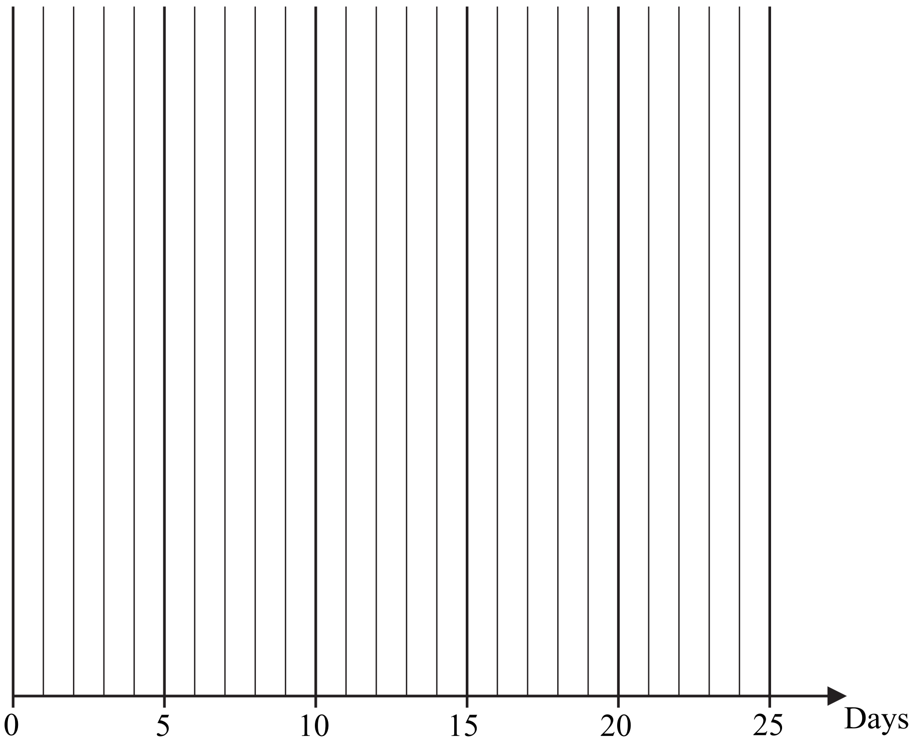
Minimum completion time is .... days.

Do not write outside the box

QUESTION PART REFERENCE

(c)

Figure 2



Turn over ▶

2 Five students attempted five different games, and penalty points were given for any mistakes that they made. The table shows the penalty points incurred by the students.

	Game 1	Game 2	Game 3	Game 4	Game 5
Ali	5	7	3	8	8
Beth	8	6	4	8	7
Cat	6	1	2	10	3
Di	4	4	3	10	7
Ell	4	6	4	7	9

Using the Hungarian algorithm, each of the five students is to be allocated to a different game so that the total number of penalty points is minimised.

(a) By reducing the **rows first** and then the columns, show that the new table of values is

2	4	0	2	3
4	2	0	1	1
5	0	1	k	0
1	1	0	4	2
0	2	0	0	3

and state the value of the constant k. *(3 marks)*

(b) Show that the zeros in the table in part **(a)** can be covered with three lines, and use augmentation to produce a table where five lines are required to cover the zeros. *(3 marks)*

(c) Hence find the possible ways of allocating the five students to the five games with the minimum total of penalty points. *(3 marks)*

(d) Find the minimum possible total of penalty points. *(1 mark)*

QUESTION PART REFERENCE

3 (a) Given that k is a constant, display the following linear programming problem in a Simplex tableau.

$$\text{Maximise} \quad P = 6x + 5y + 3z$$

$$\text{subject to} \quad x + 2y + kz \leqslant 8$$
$$2x + 10y + z \leqslant 17$$
$$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$$

(3 marks)

(b) (i) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the x-column as pivot. *(4 marks)*

(ii) Given that the maximum value of P has not been achieved after this first iteration, find the range of possible values of k. *(2 marks)*

(c) In the case where $k = -1$, perform one further iteration and interpret your final tableau. *(6 marks)*

QUESTION PART REFERENCE

4 Two people, Roger and Corrie, play a zero-sum game.

The game is represented by the following pay-off matrix for Roger.

Corrie

	Strategy	**C_1**	**C_2**	**C_3**
Roger	**R_1**	7	3	−5
	R_2	−2	−1	4

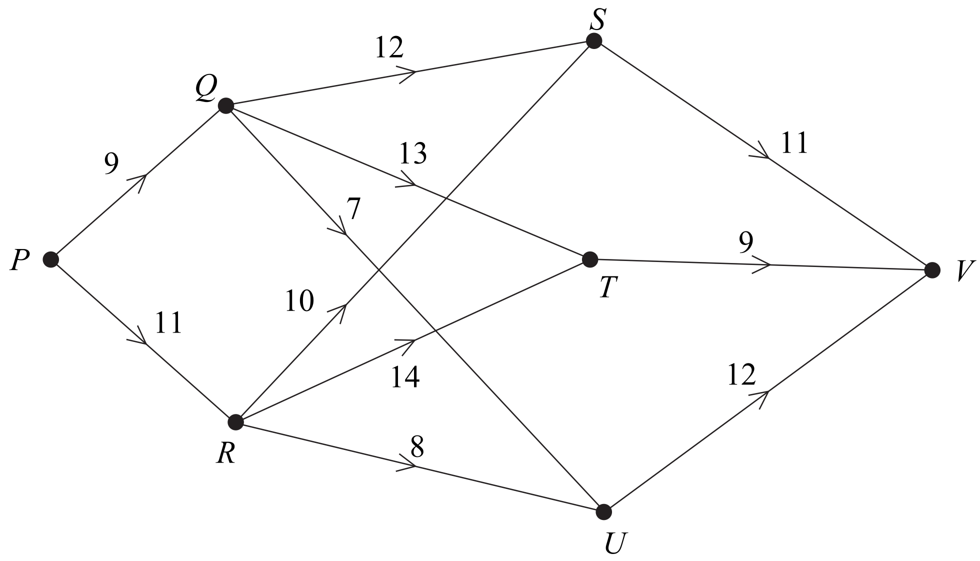
(a) (i) Find the optimal mixed strategy for Roger. *(7 marks)*

(ii) Show that the value of the game is $\frac{7}{13}$. *(1 mark)*

(b) Given that the value of the game is $\frac{7}{13}$, find the optimal mixed strategy for Corrie. *(5 marks)*

QUESTION PART REFERENCE

5 A three-day journey is to be made from P to V, with overnight stops at the end of the first day at one of the locations Q or R, and at the end of the second day at S, T or U. The network shows the journey times, in hours, for each day of the journey.



The optimal route, known as the minimax route, is that in which the longest day's journey is as small as possible.

(a) Explain why the route $PQSV$ is better than the route $PQTV$. *(2 marks)*

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from V**, to find the optimal (minimax) route from P to V.

You should indicate the calculations as well as the values at stages 2 and 3. *(8 marks)*

QUESTION PART REFERENCE

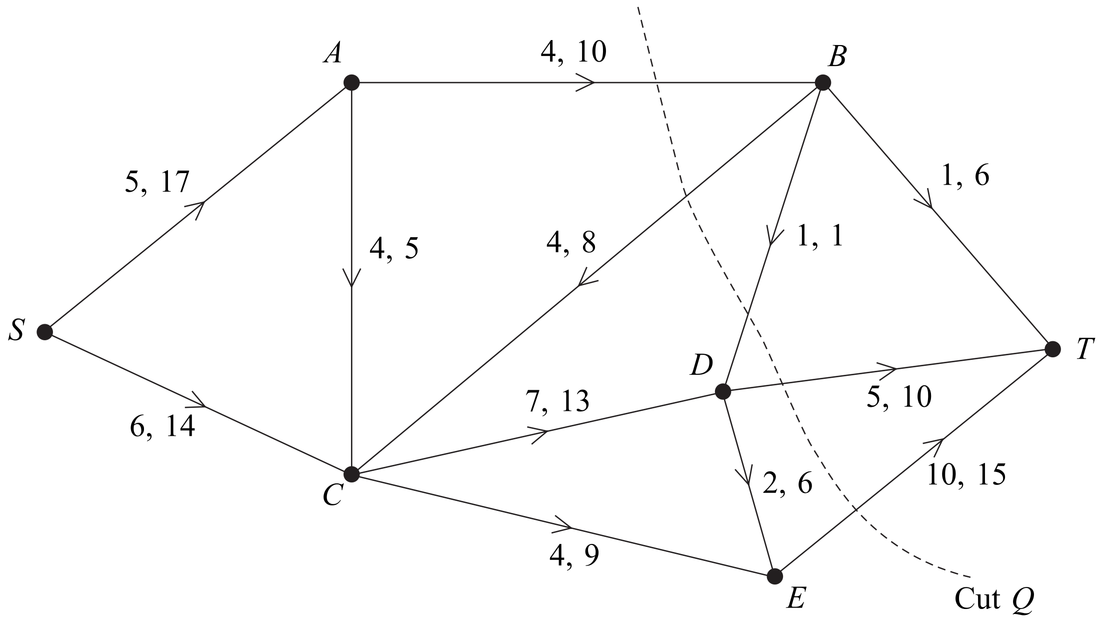
QUESTION PART REFERENCE

(b)

Stage	State	Action	Calculation	Value
1	S	SV	–	
	T	TV	–	
	U	UV	–	
2	Q	QS		

Minimax route from P to V is ....

6 The network shows a system of pipes with the lower and upper capacities for each pipe in litres per minute.



(a) Find the value of the cut Q. *(2 marks)*

(b) **Figure 3** opposite shows a partially completed diagram for a feasible flow of 24 litres per minute from S to T. Indicate, on **Figure 3**, the flows along the edges BT, DE and ET. *(2 marks)*

(c) (i) Taking your answer from part **(b)** as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 4** opposite. *(2 marks)*

(ii) Use flow augmentation on **Figure 4** to find the maximum flow from S to T.

You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 5** opposite. *(2 marks)*

(d) Find a cut with value equal to that of the maximum flow.

You may wish to show the cut on the network above. *(1 mark)*

QUESTION PART REFERENCE

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Figure 3

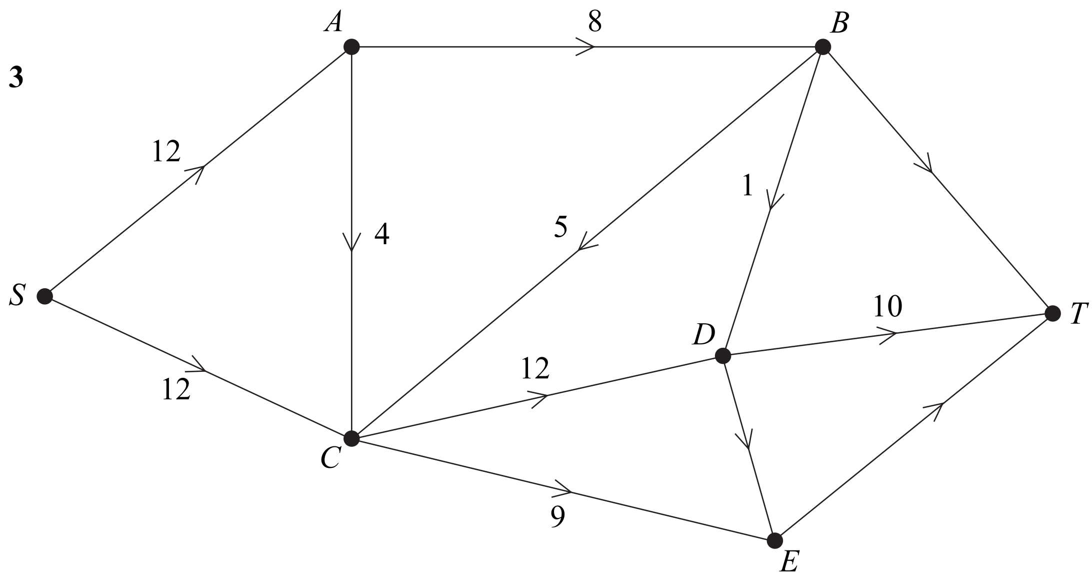
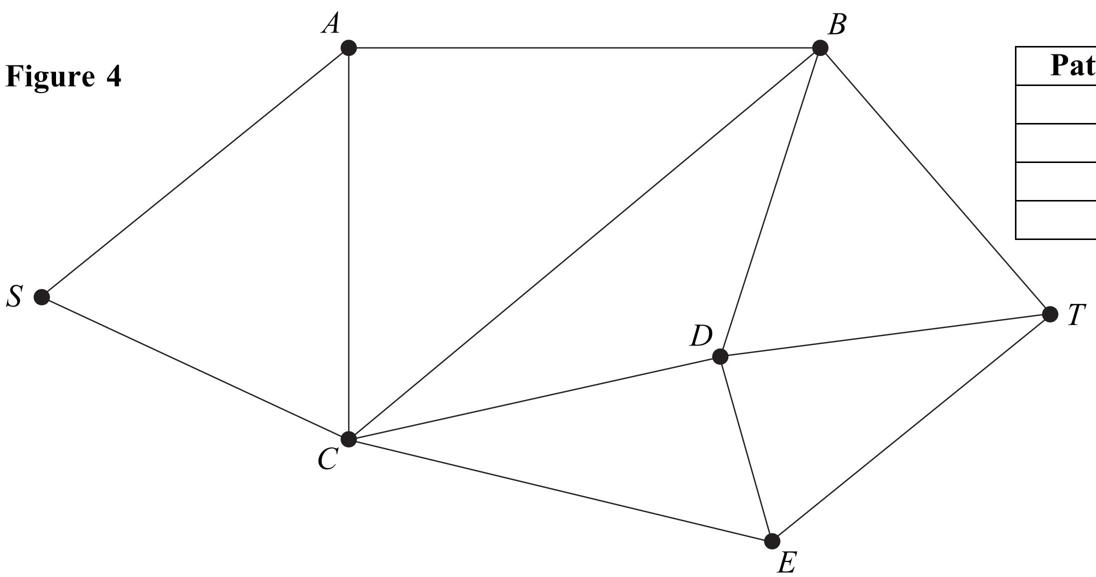
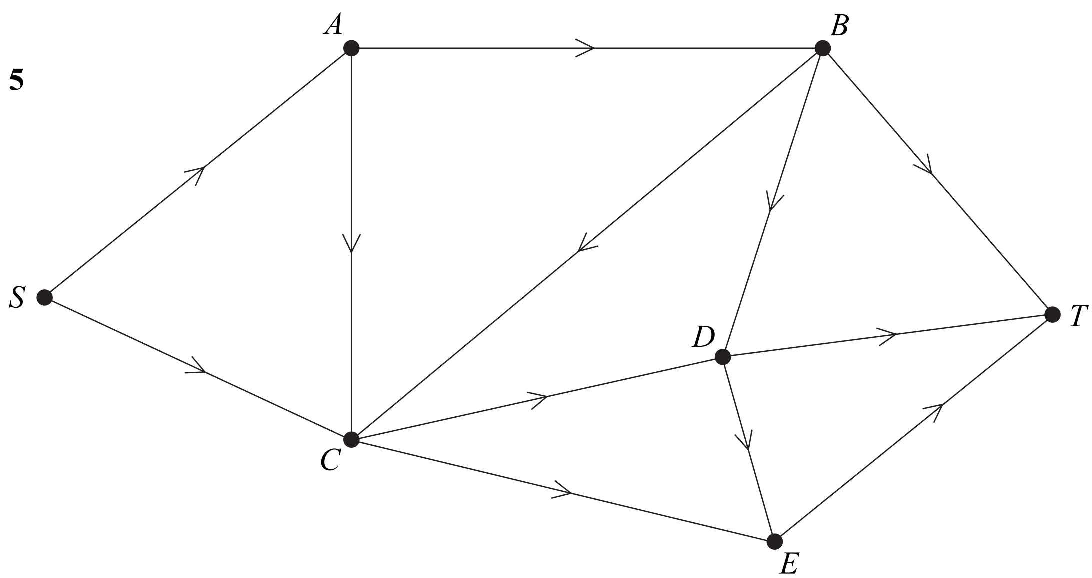


Figure 4



Path	Flow

Figure 5



Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2011

Mathematics MD02

Unit Decision 2

Wednesday 26 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

J A N 1 1 M D 0 2 0 1

P38404/Jan11/MD02 6/6/6/

Answer **all** questions in the spaces provided.

1 A group of workers is involved in a decorating project. The table shows the activities involved. Each worker can perform any of the given activities.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Duration (days)	2	5	6	7	9	4	3	2	3	2	3	1
Number of workers required	6	3	5	2	5	2	4	4	5	3	2	4

The activity network for the project is given in **Figure 1** below.

(a) Find the earliest start time and the latest finish time for each activity, inserting their values on **Figure 1**. *(4 marks)*

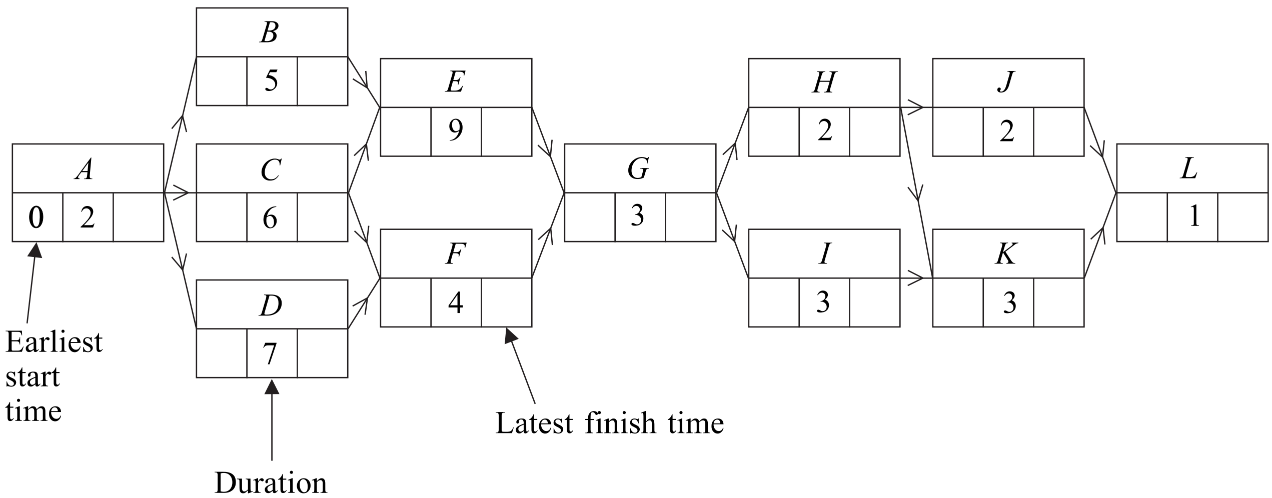
(b) Hence find:

(i) the critical path;

(ii) the float time for activity D. *(3 marks)*

(a)

Figure 1



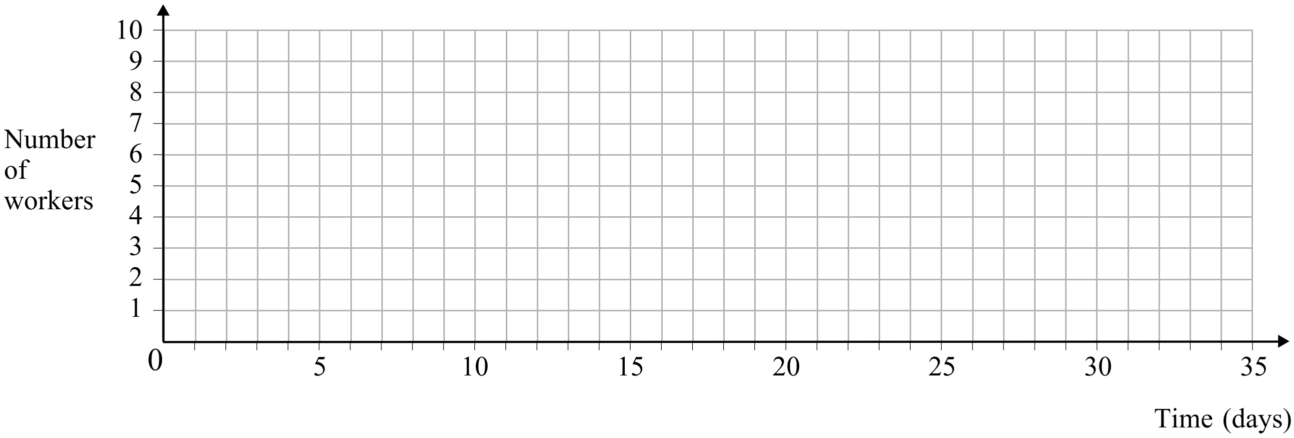
(b) (i) The critical path is ....

(ii) The float time for activity D is ....

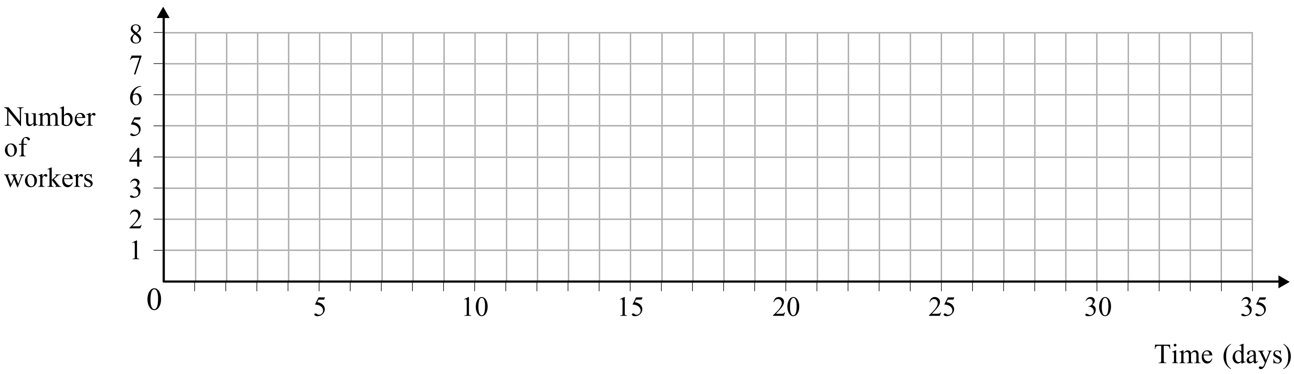
(c) Given that each activity starts as early as possible and assuming that there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2** below, indicating clearly which activities are taking place at any given time. *(4 marks)*

(d) It is later discovered that there are only 8 workers available at any time. Use resource levelling to construct a new resource histogram on **Figure 3** below, showing how the project can be completed with the minimum extra time. State the minimum extra time required. *(3 marks)*

(c) **Figure 2**



(d) **Figure 3**



The minimum extra time required is ....

2 A farmer has five fields. He intends to grow a different crop in each of four fields and to leave one of the fields unused. The farmer tests the soil in each field and calculates a score for growing each of the four crops. The scores are given in the table below.

	Field A	**Field B**	**Field C**	**Field D**	**Field E**
Crop 1	16	12	8	18	14
Crop 2	20	15	8	16	12
Crop 3	9	10	12	17	12
Crop 4	18	11	17	15	19

The farmer's aim is to maximise the total score for the four crops.

(a) (i) Modify the table of values by first subtracting each value in the table above from 20 and then adding an extra row of equal values. *(1 mark)*

(ii) Explain why the Hungarian algorithm can now be applied to the new table of values to maximise the total score for the four crops. *(3 marks)*

(b) (i) By reducing **rows** first, show that the table from part **(a)(i)** becomes

2	6	10	0	p
0	5	12	4	8
8	7	5	0	q
1	8	2	4	0
0	0	0	0	0

State the values of the constants p and q. *(2 marks)*

(ii) Show that the zeros in the table from part **(b)(i)** can be covered by one horizontal and three vertical lines, and use the Hungarian algorithm to decide how the four crops should be allocated to the fields. *(6 marks)*

(iii) Hence find the maximum possible total score for the four crops. *(1 mark)*

QUESTION PART REFERENCE

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3 Two people, Rhona and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rhona.

Colleen

	Strategy	**C_1**	**C_2**	**C_3**
	R_1	2	6	4
Rhona	**R_2**	3	-3	-1
	R_3	x	$x+3$	3

It is given that $x < 2$.

(a) (i) Write down the three row minima. *(1 mark)*

(ii) Show that there is no stable solution. *(3 marks)*

(b) Explain why Rhona should never play strategy R_3. *(1 mark)*

(c) (i) Find the optimal mixed strategy for Rhona. *(7 marks)*

(ii) Find the value of the game. *(1 mark)*

QUESTION PART REFERENCE

4 The Simplex method is to be used to maximise $P = 3x + 2y + z$ subject to the constraints

$$-x + y + z \leqslant 4$$

$$2x + y + 4z \leqslant 10$$

$$4x + 2y + 3z \leqslant 21$$

The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−3	−2	−1	0	0	0	0
0	−1	1	1	1	0	0	4
0	2	1	4	0	1	0	10
0	4	2	3	0	0	1	21

(a) (i) The first pivot is to be chosen from the x-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

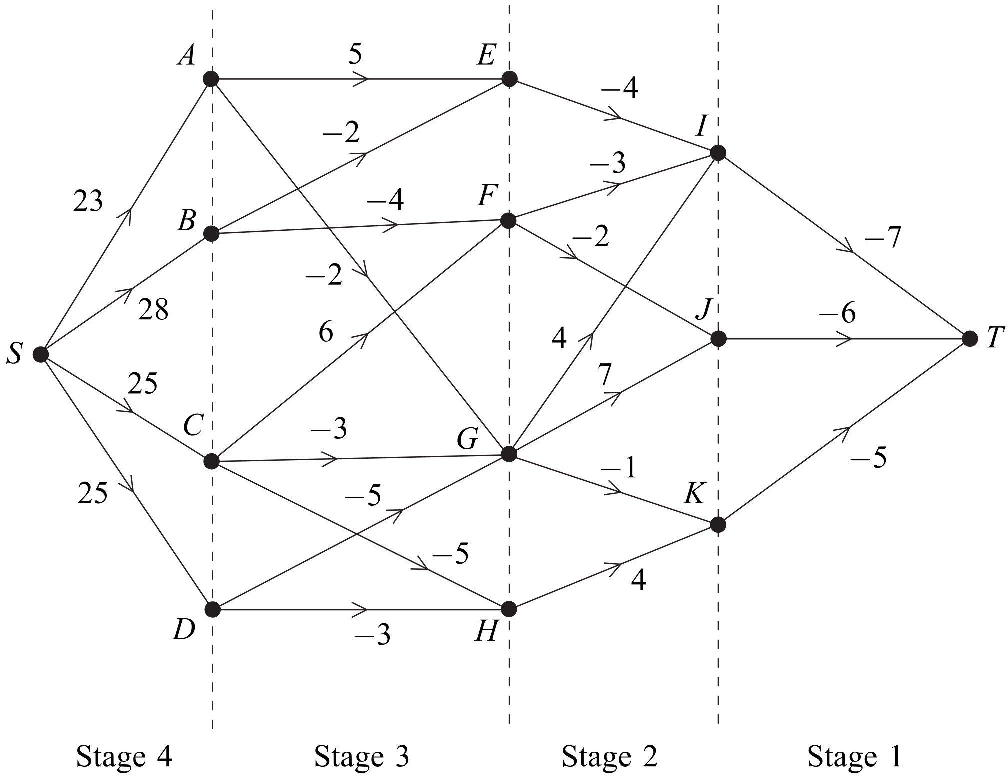
(ii) Perform one iteration of the Simplex method and explain how you know that the optimal value has not been reached. *(5 marks)*

(b) (i) Perform one further iteration. *(4 marks)*

(ii) Interpret the final tableau and write down the initial inequality that still has slack. *(4 marks)*

QUESTION PART REFERENCE

5 Each path from S to T in the network below represents a possible way of using the internet to buy a ticket for a particular event. The number on each edge represents a charge, in pounds, with a negative value representing a discount. For example, the path $SAEIT$ represents a ticket costing $23 + 5 - 4 - 7 = 17$ pounds.



(a) **By working backwards from T** and completing the table on **Figure 4**, use dynamic programming to find the minimum weight of all paths from S to T. *(6 marks)*

(b) State the minimum cost of a ticket for the event and the paths corresponding to this minimum cost. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

(a)

Figure 4

Stage	State	From	Value	
1	I	T	-7	
	J	T	-6	
	K	T	-5	
2	E	I	$-7 - 4 = -11$	
	F	I		
		J		
	G	I		
		J		
		K		
	H	K		
3				

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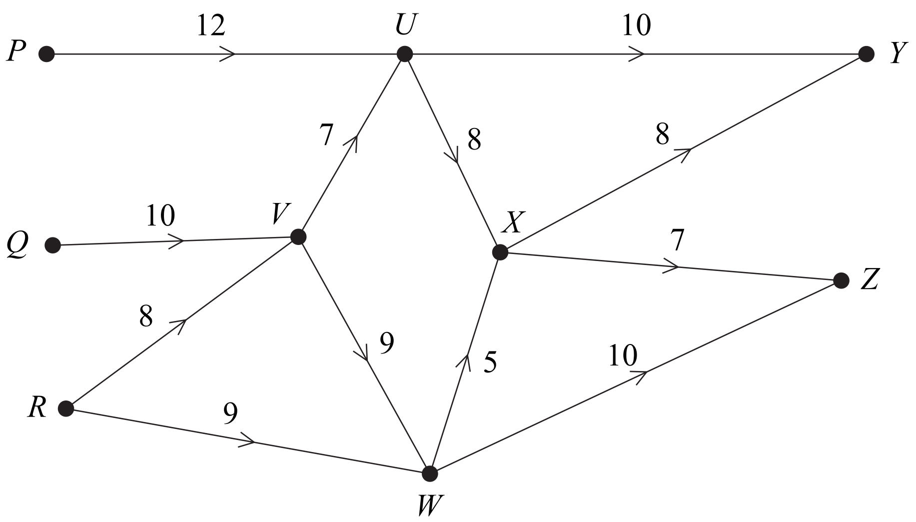
6 A retail company has warehouses at P, Q and R, and goods are to be transported to retail outlets at Y and Z. There are also retaining depots at U, V, W and X.

The possible routes with the capacities along each edge, in van loads per week, are shown in the following diagram.



(a) On **Figure 5 opposite**, add a super-source, S, and a super-sink, T, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each edge that you have added. *(2 marks)*

(b) On **Figure 6**, write down the maximum flows along the routes $SPUYT$ and $SRVWZT$. *(2 marks)*

(c) (i) On **Figure 7**, add the vertices S and T and the edges connecting S and T to the network. Using the maximum flows along the routes $SPUYT$ and $SRVWZT$ found in part **(b)** as the initial flow, indicate the potential increases and decreases of the flow on each edge of these routes. *(2 marks)*

(ii) Use flow augmentation to find the maximum flow from S to T. You should indicate any flow-augmenting routes on **Figure 6** and modify the potential increases and decreases of the flow on **Figure 7**. *(4 marks)*

(d) Find a cut with value equal to the maximum flow. *(1 mark)*

QUESTION PART REFERENCE

Figure 5

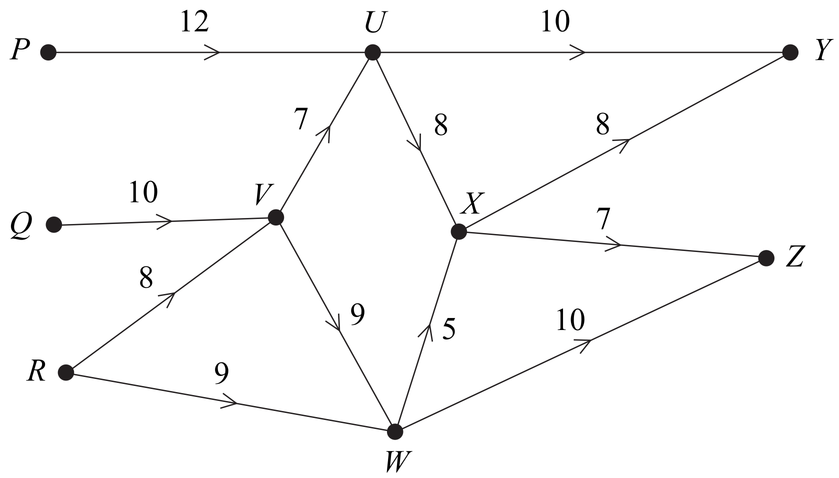
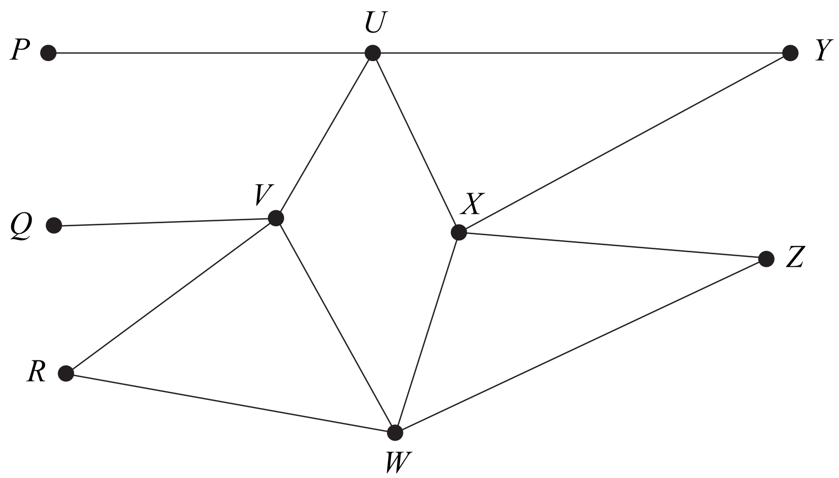


Figure 6

Route	Flow
SPUYT	
SRVWZT	

Figure 7



Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
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3	
4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MD02

Unit Decision 2

Monday 20 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 **Figure 1** below shows an activity diagram for a cleaning project. The duration of each activity is given in days.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical paths and state the minimum time for completion of the project. *(3 marks)*

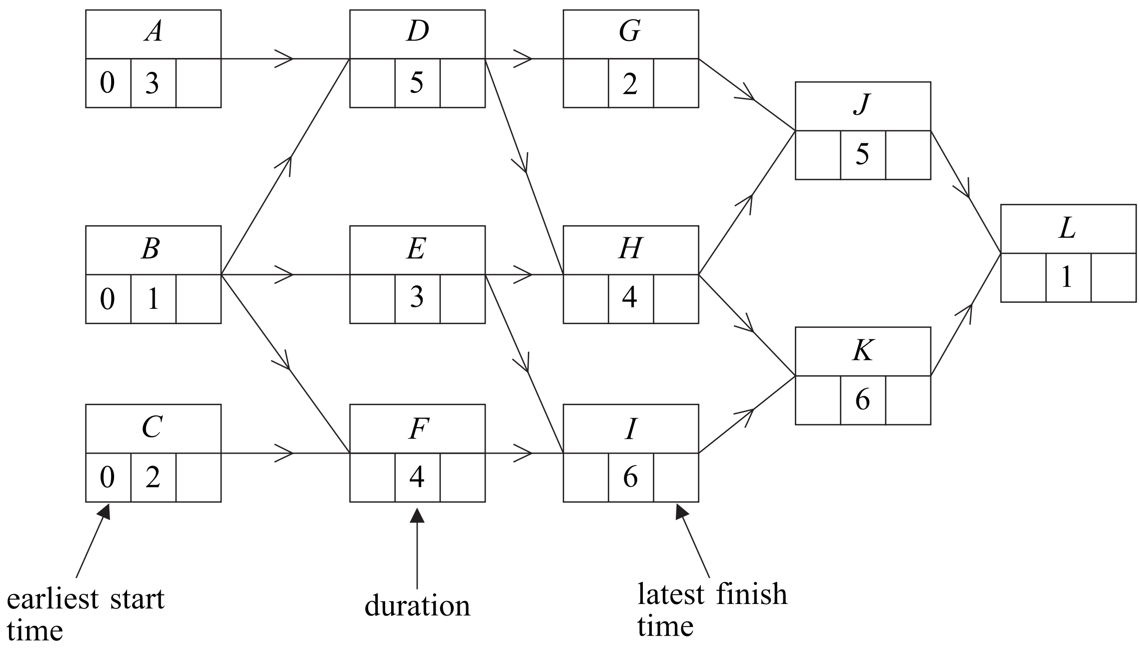
(c) Find the activity with the greatest float time and state the value of its float time. *(2 marks)*

(d) On **Figure 2** opposite, draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as **late** as possible. *(4 marks)*

QUESTION PART REFERENCE

(a)

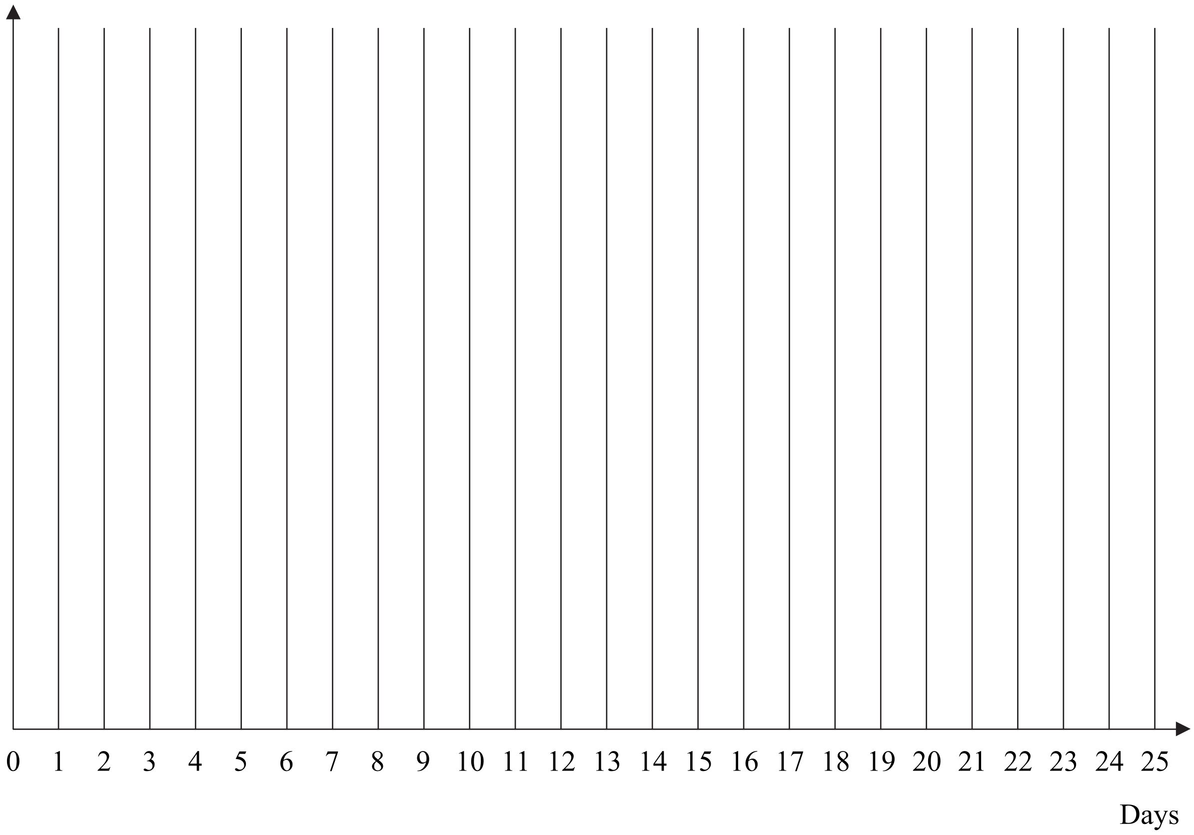
Figure 1



QUESTION PART REFERENCE

(d)

Figure 2



2 The times taken, in minutes, for five people, A, B, C, D and E, to complete each of five different puzzles are recorded in the table below.

	A	B	C	D	E
Puzzle 1	16	13	15	16	15
Puzzle 2	14	16	16	14	18
Puzzle 3	14	12	18	13	16
Puzzle 4	15	15	17	12	14
Puzzle 5	13	17	16	14	15

Using the Hungarian algorithm, each of the five people is to be allocated to a different puzzle so that the total time for completing the five puzzles is minimised.

(a) By reducing the **columns first** and then the rows, show that the new table of values is

3	1	0	4	1
0	k	0	1	3
1	0	3	1	2
2	3	2	0	0
0	5	1	2	1

State the value of the constant k. *(2 marks)*

(b) (i) Show that the zeros in the table in part **(a)** can be covered with one horizontal and three vertical lines. *(1 mark)*

(ii) Use augmentation to produce a table where five lines are required to cover the zeros. *(2 marks)*

(c) Hence find all the possible ways of allocating the five people to the five puzzles so that the total completion time is minimised. *(3 marks)*

(d) Find the minimum total time for completing the five puzzles. *(1 mark)*

(e) Explain how you would modify the original table if the Hungarian algorithm were to be used to find the **maximum** total time for completing the five puzzles using five different people. *(1 mark)*

QUESTION PART REFERENCE

3 (a) Two people, Tom and Jerry, play a zero-sum game. The game is represented by the following pay-off matrix for Tom.

Jerry

	Strategy	**A**	**B**	**C**
Tom	**I**	−4	5	−3
	II	−3	−2	8
	III	−7	6	−2

Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) Rohan and Carla play a different zero-sum game for which there is no stable solution. The game is represented by the following pay-off matrix for Rohan.

Carla

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Rohan	$\mathbf{R_1}$	3	5	−1
	$\mathbf{R_2}$	1	−2	4

(i) Find the optimal mixed strategy for Rohan and show that the value of the game is $\frac{3}{2}$. *(7 marks)*

(ii) Carla plays strategy C_1 with probability p, and strategy C_2 with probability q.

Find the values of p and q and hence find the optimal mixed strategy for Carla. *(4 marks)*

QUESTION PART REFERENCE

4 A linear programming problem involving variables x, y and z is to be solved. The objective function to be maximised is $P = 2x + 6y + kz$, where k is a constant.

The initial Simplex tableau is given below.

P	***x***	***y***	***z***	***s***	***t***	***u***	***value***
1	−2	−6	$-k$	0	0	0	0
0	5	3	10	1	0	0	15
0	7	6	4	0	1	0	28
0	4	3	6	0	0	1	12

(a) In addition to $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$, write down **three** inequalities involving x, y and z for this problem. *(2 marks)*

(b) (i) By choosing the first pivot **from the *y*-column**, perform **one** iteration of the Simplex method. *(4 marks)*

(ii) Given that the optimal value has **not** been reached, find the possible values of k. *(2 marks)*

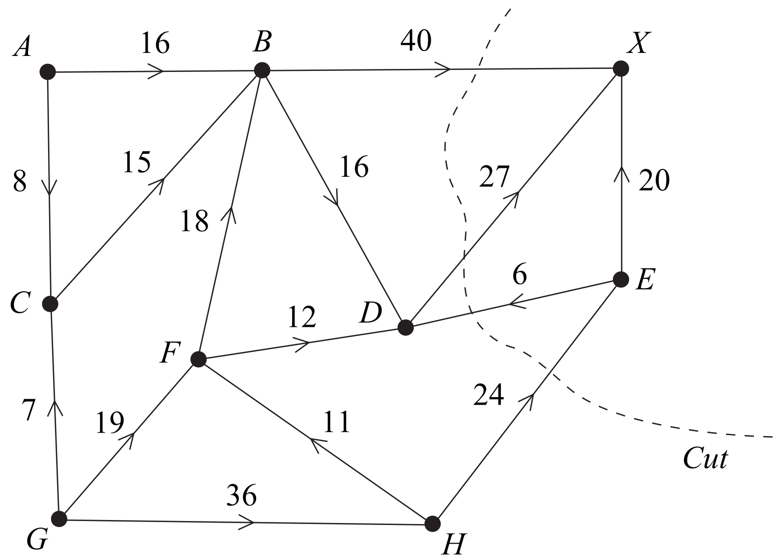
(c) In the case when $k = 20$:

(i) perform one further iteration; *(4 marks)*

(ii) interpret the final tableau and state the values of the slack variables. *(3 marks)*

QUESTION PART REFERENCE

5 The network shows the evacuation routes along corridors in a college, from two teaching areas to the exit, in case of a fire alarm sounding.



The two teaching areas are at A and G and the exit is at X.

The number on each edge represents the maximum number of people that can travel along a particular corridor in one minute.

(a) Find the value of the cut shown on the diagram. *(1 mark)*

(b) Find the maximum flow along each of the routes *ABDX*, *GFBX* and *GHEX* and enter their values in the table on **Figure 3** opposite. *(3 marks)*

(c) (i) Taking your answers to part **(b)** as the initial flow, use the labelling procedure on **Figure 3** to find the maximum flow through the network. You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

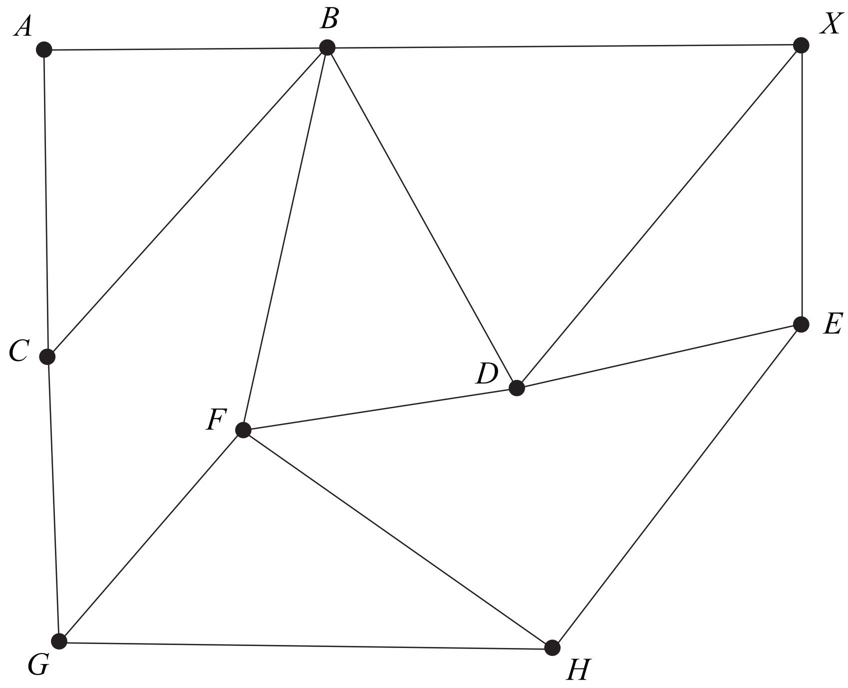
(ii) State the value of the maximum flow, and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(d) During one particular fire drill, there is an obstruction allowing no more than 45 people per minute to pass through vertex B. State the maximum number of people that can move through the network per minute during this fire drill. *(2 marks)*

QUESTION PART REFERENCE

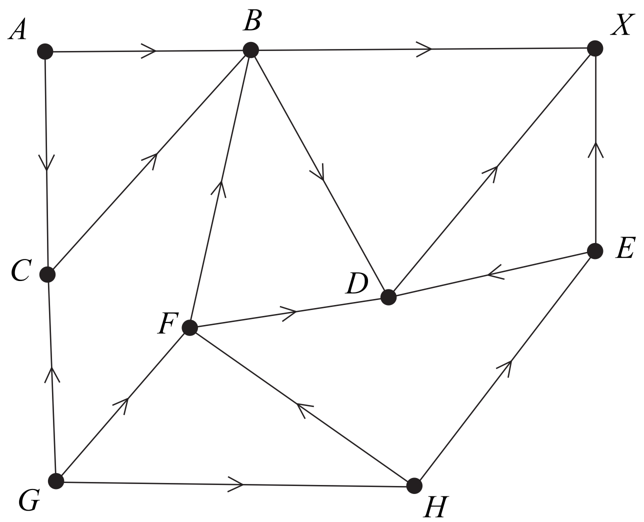
QUESTION PART REFERENCE

Figure 3



Route	Flow
ABDX	
GFBX	
GHEX	

Figure 4



Maximum flow is .... people per minute.

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Turn over ▶

6 Bob is planning to build four garden sheds, A, B, C and D, at the rate of one per day. The order in which they are built is a matter of choice, but the costs will vary because some of the materials left over from making one shed can be used for the next one. The expected profits, in pounds, are given in the table below.

Day	Already built	Expected profit (£)			
		A	***B***	***C***	***D***
Monday	–	50	65	70	80
Tuesday	***A***	–	72	83	84
	B	60	–	80	83
	C	57	68	–	85
	D	62	70	81	–
Wednesday	***A* and *B***	–	–	84	88
	A* and *C	–	71	–	82
	A* and *D	–	74	83	–
	B* and *C	65	–	–	86
	B* and *D	69	–	85	–
	C* and *D	66	73	–	–
Thursday	***A*, *B* and *C***	–	–	–	90
	A*, *B* and *D	–	–	87	–
	A*, *C* and *D	–	76	–	–
	B*, *C* and *D	70	–	–	–

By completing the table of values opposite, or otherwise, use dynamic programming, **working backwards from Thursday**, to find the building schedule that maximises the total expected profit. *(9 marks)*

QUESTION PART REFERENCE

Stage (Day)	State (Sheds already built)	Action (Shed to build)	Calculation	Profit in pounds
Thursday	A, B, C	D		90
	A, B, D	C		87
	A, C, D	B		76
	B, C, D	A		70
Wednesday	A, B	C	$84 + 90$	174
		D	$88 + 87$	175
	A, C	B	$71 + 90$	161
		D	$82 + 76$	158
	A, D	B		
		C		
	B, C	A		
		D		
	B, D	A		
		C		
	C, D	A		
		B		
Tuesday	A	B	$72 + 175$	247
		C	$83 + 161$	244
		D		
Monday				

Schedule

	Monday	Tuesday	Wednesday	Thursday
Shed to build				

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MD02

Unit Decision 2

Wednesday 25 January 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

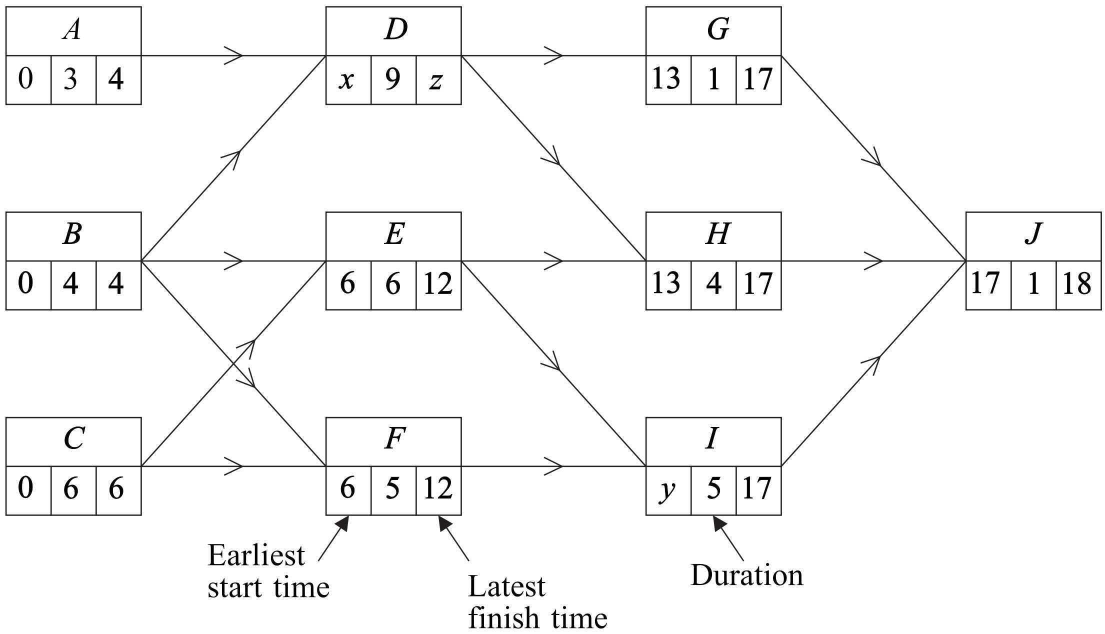
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 The diagram shows the activity network and the duration, in days, of each activity for a particular project. Some of the earliest start times and latest finish times are shown on the diagram.



(a) Find the values of the constants x, y and z. *(3 marks)*

(b) Find the critical paths. *(2 marks)*

(c) Find the activity with the largest float and state the value of this float. *(2 marks)*

QUESTION PART REFERENCE

(d) The number of workers required for each activity is shown in the table.

Activity	A	B	C	D	E	F	G	H	I	J
Number of workers required	4	2	3	4	2	4	3	3	5	6

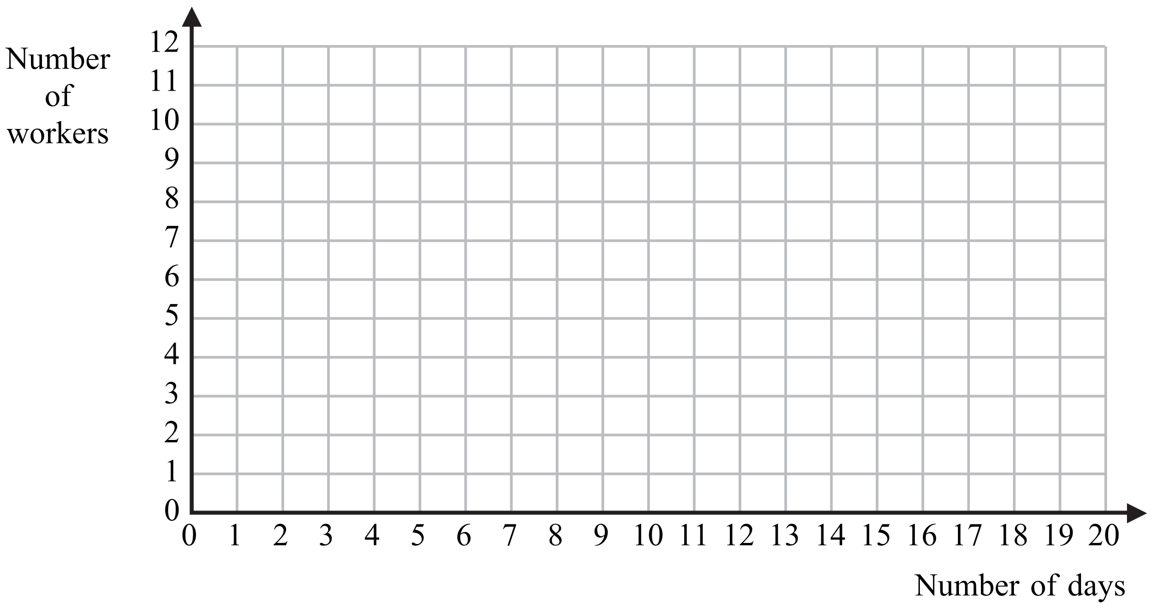
Given that each activity starts as **early** as possible and assuming that there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 1** below, indicating clearly which activities are taking place at any given time. *(5 marks)*

(e) It is later discovered that there are only 9 workers available at any time. Use resource levelling to find the new earliest start time for activity J so that the project can be completed with the minimum extra time. State the minimum extra time required. *(2 marks)*

QUESTION PART REFERENCE

(d)

Figure 1



2 A team with five members is training to take part in a quiz. The team members, Abby, Bob, Cait, Drew and Ellie, attempted sample questions on each of the five topics and their scores are given in the table.

	Topic 1	**Topic 2**	**Topic 3**	**Topic 4**	**Topic 5**
Abby	27	29	25	35	31
Bob	33	22	17	29	29
Cait	23	29	25	33	21
Drew	22	29	29	27	31
Ellie	27	27	19	21	27

For the actual quiz, each topic must be allocated to exactly one of the team members. The maximum total score for the sample questions is to be used to allocate the different topics to the team members.

(a) Explain why the Hungarian algorithm may be used if each number, x, in the table is replaced by $35 - x$. *(2 marks)*

(b) Form a new table by subtracting each number in the table above from 35. Hence show that, by reducing **rows first** then columns, the resulting table of values is as below, stating the values of the constants p and q.

8	6	8	0	4
0	11	p	4	4
10	4	6	0	12
q	2	0	4	0
0	0	6	6	0

(3 marks)

(c) Show that the zeros in the table in part **(b)** can be covered with two horizontal and two vertical lines. Hence use the Hungarian algorithm to reduce the table to a form where five lines are needed to cover the zeros. *(3 marks)*

(d) (i) Hence find the possible allocations of topics to the five team members so that the total score for the sample questions is maximised. *(3 marks)*

(ii) State the value of this maximum total score. *(1 mark)*

3 Two people, Roz and Colum, play a zero-sum game. The game is represented by the following pay-off matrix for Roz.

Colum

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
	$\mathbf{R_1}$	-2	-6	-1
Roz	$\mathbf{R_2}$	-5	2	-6
	$\mathbf{R_3}$	-3	3	-4

(a) Explain what is meant by the term 'zero-sum game'. *(2 marks)*

(b) Determine the play-safe strategy for Colum, giving a reason for your answer. *(2 marks)*

(c) (i) Show that the matrix can be reduced to a 2 by 3 matrix, giving the reason for deleting one of the rows. *(2 marks)*

(ii) Hence find the optimal mixed strategy for Roz. *(7 marks)*

QUESTION PART REFERENCE

4 A linear programming problem consists of maximising an objective function P involving three variables, x, y and z, subject to constraints given by three inequalities other than $x \geqslant 0$, $y \geqslant 0$ and $z \geqslant 0$. Slack variables s, t and u are introduced and the Simplex method is used to solve the problem. One iteration of the method leads to the following tableau.

P	x	y	z	s	t	u	*value*
1	−2	11	0	3	0	0	6
0	2	3	1	1	0	0	2
0	6	−30	0	−6	1	0	3
0	−1	−9	0	−3	0	1	4

(a) (i) State the column from which the pivot for the **next** iteration should be chosen. Identify this pivot and explain the reason for your choice. *(3 marks)*

(ii) Perform the next iteration of the Simplex method. *(4 marks)*

(b) (i) Explain why you know that the maximum value of P has been achieved. *(1 mark)*

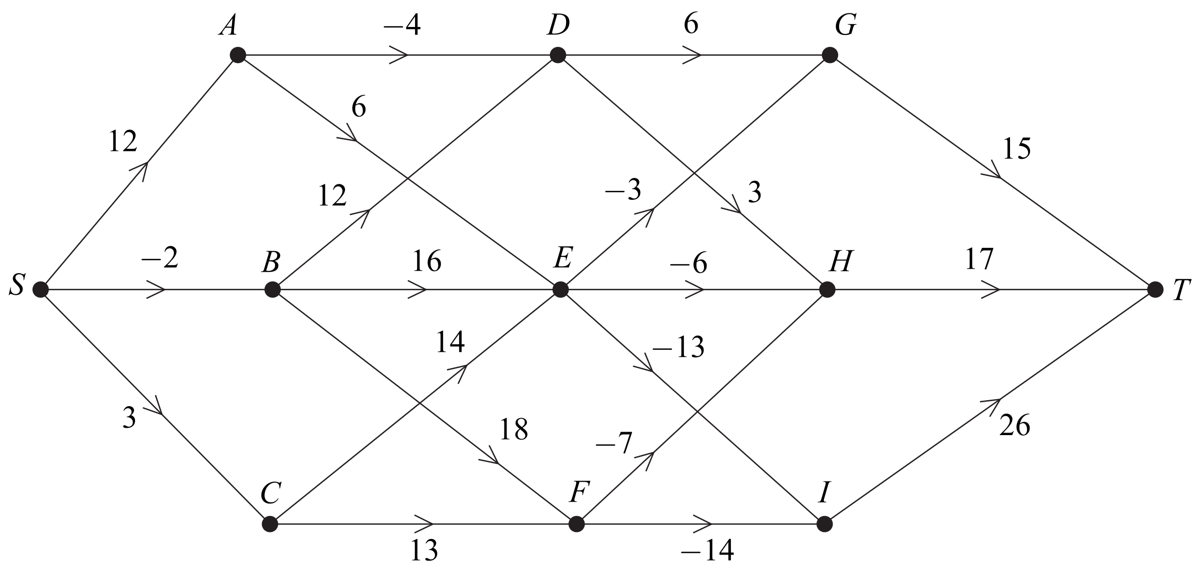
(ii) State how many of the three original inequalities still have slack. *(1 mark)*

(c) (i) State the maximum value of P and the values of x, y and z that produce this maximum value. *(2 marks)*

(ii) The objective function for this problem is $P = kx - 2y + 3z$, where k is a constant. Find the value of k. *(2 marks)*

QUESTION PART REFERENCE

5 A firm is considering various strategies for development over the next few years. In the network, the number on each edge is the expected profit, in millions of pounds, moving from one year to the next. A negative number indicates a loss because of building costs or other expenses. Each path from S to T represents a complete strategy.



(a) By completing the table on the page opposite, or otherwise, use dynamic programming **working backwards from *T*** to find the maximum weight of all paths from S to T. *(6 marks)*

(b) State the overall maximum profit and the paths from S to T corresponding to this maximum profit. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

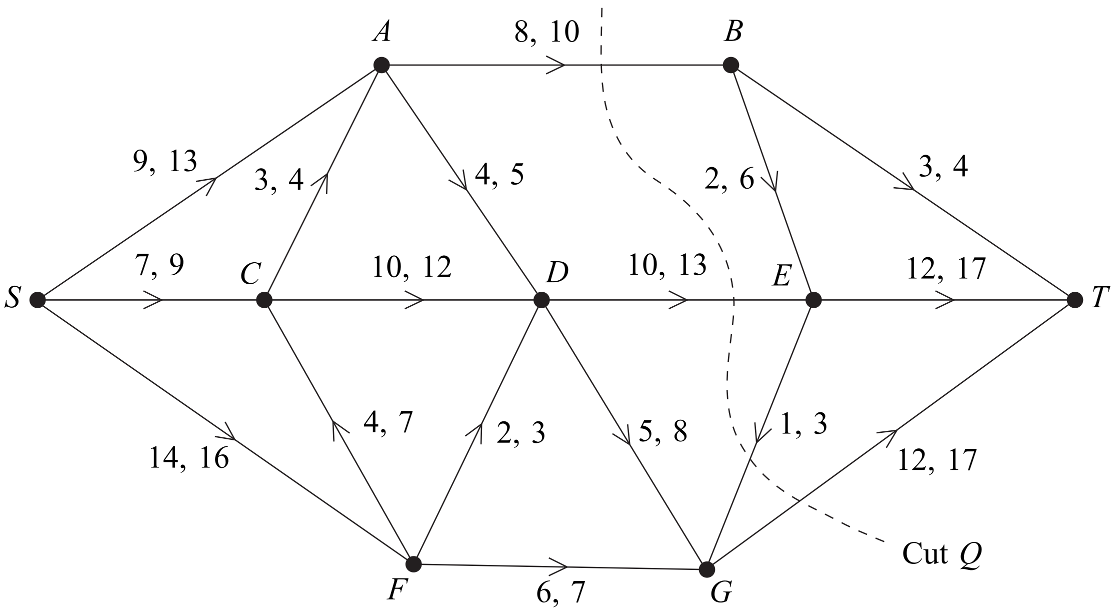
(a)

Stage	State	From	Calculation	Value
1	G	T		
	H	T		
	I	T		
2	D	G		
		H		
	E	G		
		H		
		I		
	F	H		
		I		
3				

(b) Maximum profit is £.............................. million

Corresponding paths from S to T ....

6 The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) Find the value of the cut Q. *(2 marks)*

(b) **Figure 2** shows most of the values of a feasible flow of 34 litres per second from S to T.

(i) Insert the missing values of the flows along DE and FG on **Figure 2**. *(2 marks)*

(ii) Using this feasible flow as the initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 3**. *(2 marks)*

(iii) Use flow augmentation on **Figure 3** to find the maximum flow from S to T. You should indicate any flow-augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(4 marks)*

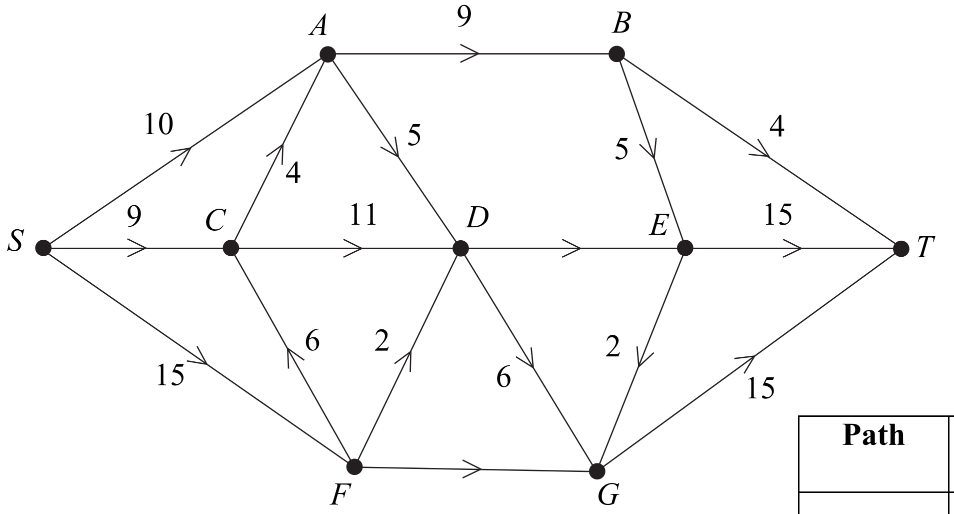
(c) (i) State the value of the maximum flow. *(1 mark)*

(ii) Illustrate your maximum flow on **Figure 4**. *(2 marks)*

(d) Find a cut with capacity equal to that of the maximum flow. *(1 mark)*

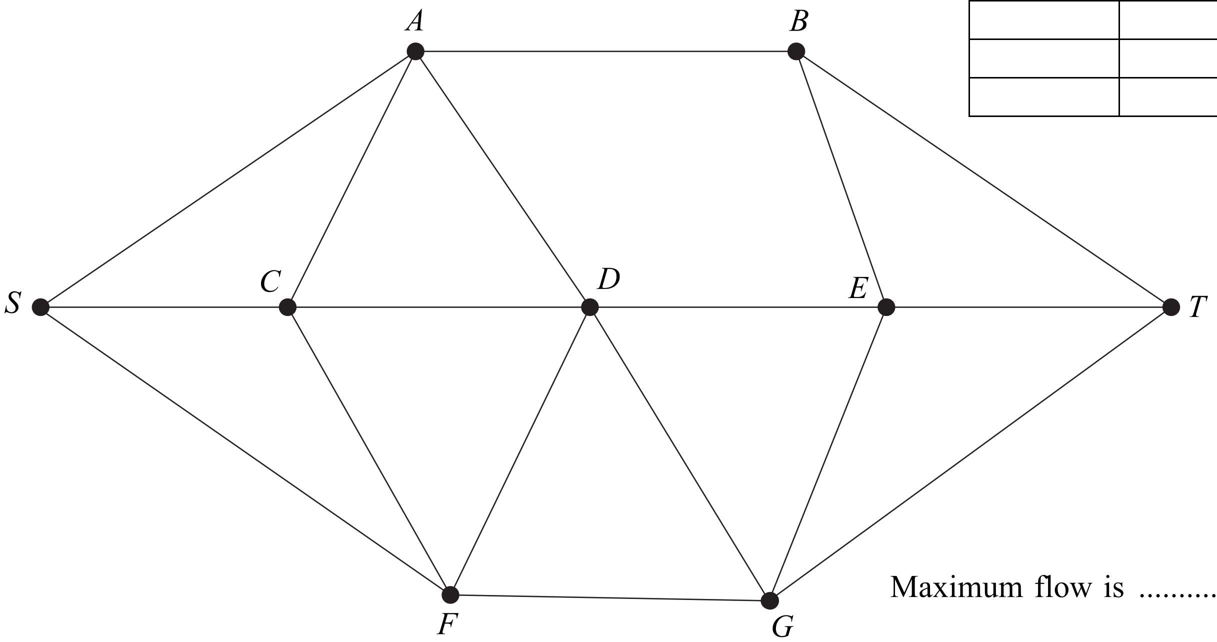
QUESTION PART REFERENCE

Figure 2



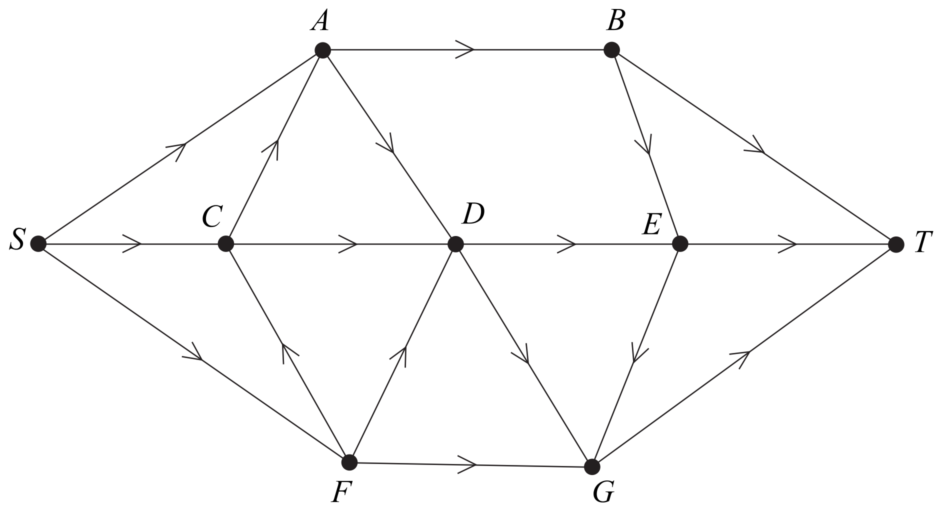
Path	Extra Flow

Figure 3



Maximum flow is ....

Figure 4



Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MD02

Unit Decision 2

Thursday 21 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** below shows an activity diagram for a construction project. The time needed for each activity is given in days.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical paths and state the minimum time for completion of the project. *(3 marks)*

(c) On **Figure 2** opposite, draw a cascade diagram (Gantt chart) for the project, assuming that each activity starts as early as possible. *(5 marks)*

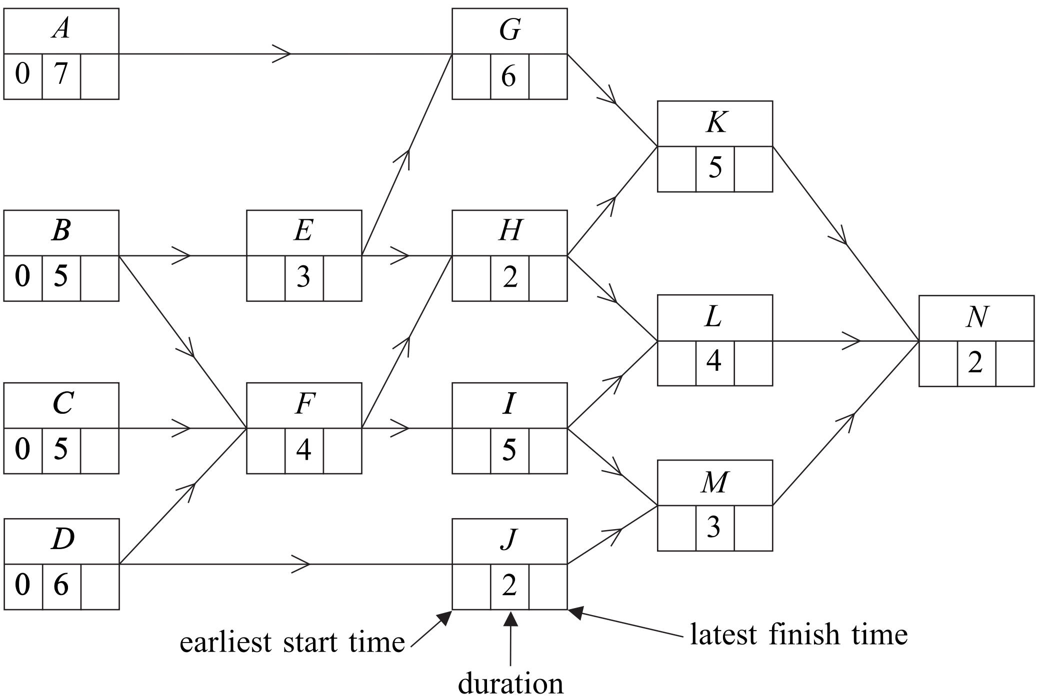
(d) Activity J takes longer than expected so that its duration is x days, where $x \geqslant 3$. Given that the minimum time for completion of the project is unchanged, find a further inequality relating to the maximum value of x. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

(a)

Figure 1



(b) Critical paths are ....

....

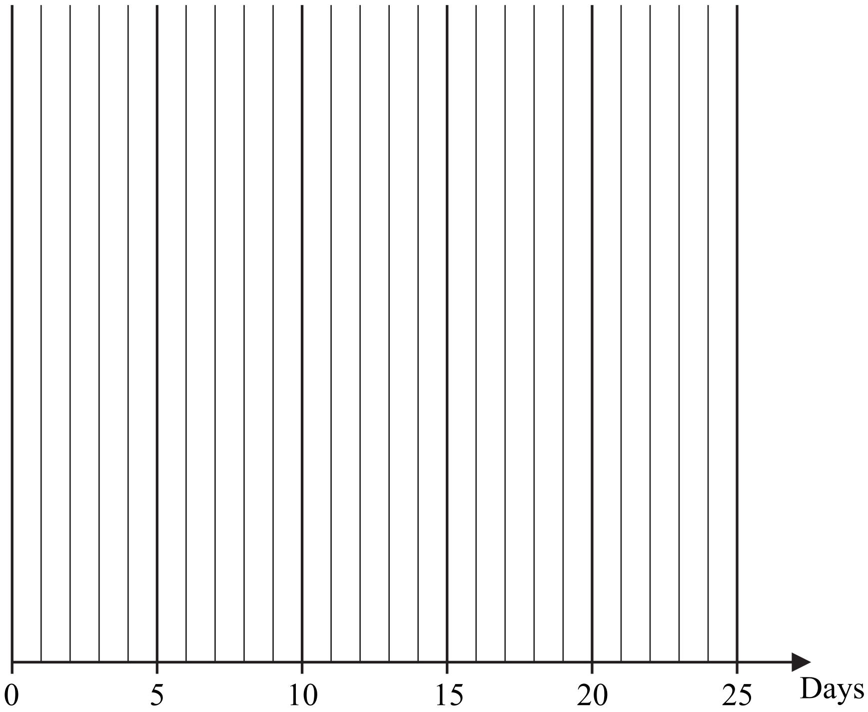
Minimum completion time is .... days.

QUESTION PART REFERENCE

Answer space for question 1

(c)

Figure 2



(d)

2 The times taken in minutes for five people, Ann, Baz, Cal, Di and Ez, to complete each of five different tasks are recorded in the table below. Neither Ann nor Di can do task 2, as indicated by the asterisks in the table.

	Ann	Baz	Cal	Di	Ez
Task 1	13	14	15	17	16
Task 2	***	21	21	***	18
Task 3	16	19	19	17	15
Task 4	16	16	18	16	16
Task 5	20	23	22	20	20

Using the Hungarian algorithm, each of the five people is to be allocated to a different task so that the total time for completing the five tasks is minimised.

(a) By reducing the **rows first** and then the columns, show that the zeros in the new table of values can be covered with four lines. *(3 marks)*

(b) Use adjustments to produce a table where five lines are required to cover the zeros. *(3 marks)*

(c) Hence find the possible ways of allocating the five people to the five tasks in the minimum total time. *(3 marks)*

(d) State the minimum total time for completing the five tasks. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

3 (a) Given that k is a constant, complete the Simplex tableau below for the following linear programming problem.

Maximise $P = kx + 6y + 5z$

subject to
$$2x + y + 4z \leqslant 11$$
$$x + 3y + 6z \leqslant 18$$
$$x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$$
(2 marks)

(b) Use the Simplex method to perform **one** iteration of your tableau for part **(a)**, choosing a value in the ***y*-column** as pivot. *(4 marks)*

(c) (i) In the case when $k = 1$, explain why the maximum value of P has now been reached and write down this maximum value of P. *(2 marks)*

(ii) In the case when $k = 3$, perform one further iteration and interpret your new tableau. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 3

(a)

P	x	y	z	s	t	*value*
1	$-k$	-6	-5	0	0	0
0						
0						

....

....

....

(b)

P	x	y	z	s	t	*value*

....

....

....

QUESTION PART REFERENCE

Answer space for question 3

(c)(i)

(c)(ii)

P	x	y	z	s	t	*value*

4 (a) Two people, Adam and Bill, play a zero-sum game. The game is represented by the following pay-off matrix for Adam.

Bill

	Strategy	**B_1**	**B_2**	**B_3**
	A_1	-6	-1	-5
	A_2	5	2	-3
Adam	**A_3**	-5	4	-4
	A_4	2	1	-4

(i) Show that this game has a stable solution. *(3 marks)*

(ii) Find the play-safe strategy for each player. *(1 mark)*

(iii) State the value of the game for **Bill**. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4(a)

Do not write outside the box

4 (b) Roza plays a different zero-sum game against a computer. The game is represented by the following pay-off matrix for Roza.

Computer

	Strategy	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$
Roza	$\mathbf{R_1}$	3	4	-3
	$\mathbf{R_2}$	-2	-1	5

(i) State which strategy the computer should never play, giving a reason for your answer. *(1 mark)*

(ii) Roza chooses strategy R_1 with probability p. Find expressions for the expected gains for Roza when the computer chooses each of its two remaining strategies. *(2 marks)*

(iii) Hence find the value of p for which Roza will maximise her expected gains. *(2 marks)*

(iv) Find the value of the game for Roza. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4(b)

5 Dave plans to renovate three houses, A, B and C, at the rate of one per year. The order in which they are renovated is a matter of choice, but some costs vary over the three years. The expected costs, in thousands of pounds, are given in the table below.

Year	Already renovated	Cost A	Cost B	Cost C
1	–	60	70	65
2	A	–	75	70
	B	55	–	60
	C	65	80	–
3	A and B	–	–	75
	A and C	–	80	–
	B and C	60	–	–

For tax reasons, Dave needs to choose the order for renovation so that the least annual cost is as large as possible. Solving the maximin problem will produce this optimum order for renovation.

(a) (i) State the least annual cost when the order of renovation is BAC.

(ii) Determine, with a reason, whether the order ABC is better than the order BAC. *(3 marks)*

(b) By completing the table opposite, or otherwise, use dynamic programming, **working backwards from Year 3**, to find the optimum order for renovation. *(7 marks)*

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

(b)

Answer space for question 5

Year	**Already renovated**	**House renovated**	**Calculation**	**Value**
3	A and B	C		
	A and C	B		
	B and C	A		
2	A	B		
		C		
	B	A		
		C		
	C	A		
		B		
1				

Optimum order ....

....

....

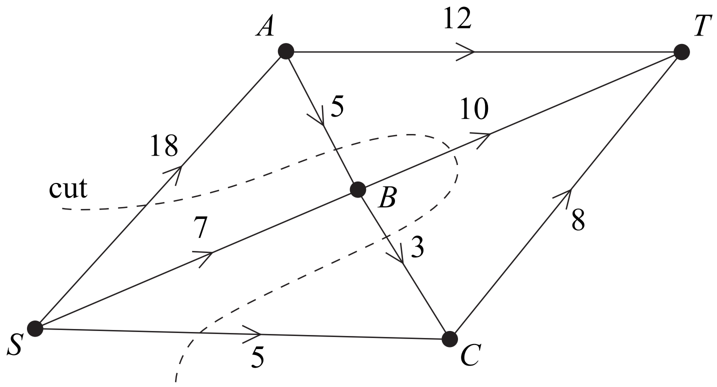
....

....

....

....

6 (a) The network shows a flow from S to T along a system of pipes, with the capacity in litres per second indicated on each edge.



(i) Show that the value of the cut shown on the diagram is 36. *(1 mark)*

....

....

(ii) The cut shown on the diagram can be represented as $\{S, B\}$, $\{A, C, T\}$.

Complete the table below to give the value of each of the 8 possible cuts. *(3 marks)*

Cut		**Value**
$\{S\}$	$\{A, B, C, T\}$	30
$\{S, A\}$	$\{B, C, T\}$	29
$\{S, B\}$	$\{A, C, T\}$	36
$\{S, C\}$	$\{A, B, T\}$	33
$\{S, A, B\}$	$\{C, T\}$	
$\{S, A, C\}$	$\{B, T\}$	
$\{S, B, C\}$	$\{A, T\}$	
$\{S, A, B, C\}$	$\{T\}$	30

(iii) State the value of the maximum flow through the network, giving a reason for your answer. *(2 marks)*

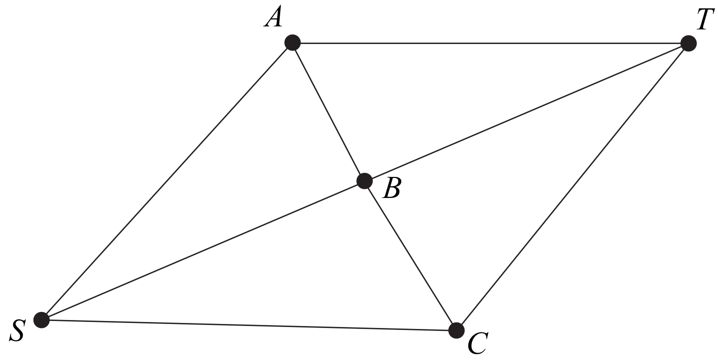
Maximum flow is ....

because ....

....

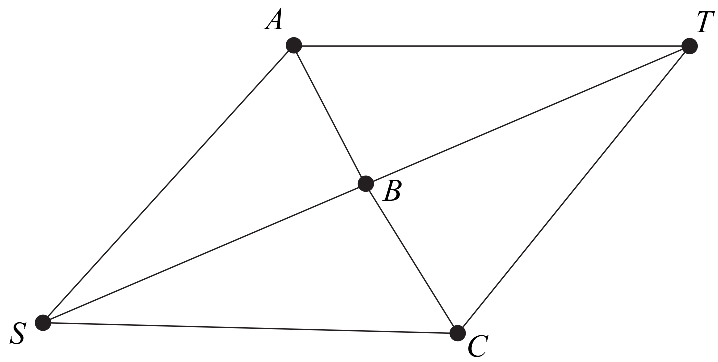
....

(iv) Indicate on the diagram below a possible flow along each edge corresponding to this maximum flow. *(1 mark)*



(b) The capacities along SC and along AT are each increased by 4 litres per second.

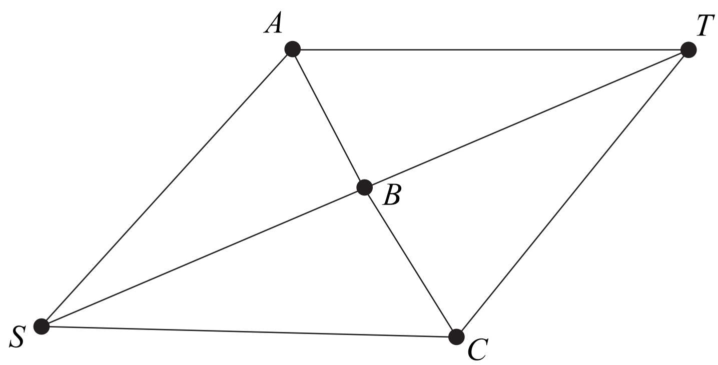
(i) Using your **values from part (a)(iv) as the initial flow**, indicate potential increases and decreases on the diagram below and use the labelling procedure to find the new maximum flow through the network. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the diagram. *(6 marks)*



Path	Additional Flow

(ii) Use your results from part **(b)(i)** to illustrate the flow along each edge that gives this new maximum flow, and state the value of the new maximum flow. *(3 marks)*

New maximum flow is ....



END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
January 2013

Mathematics MD02

Unit Decision 2

Monday 28 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** below shows an activity diagram for a project. Each activity requires one worker. The duration required for each activity is given in hours.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) On **Figure 2** opposite, complete the precedence table. *(2 marks)*

(c) Find the critical path. *(1 mark)*

(d) Find the float time of activity E. *(1 mark)*

(e) Using **Figure 3** on page 5, draw a resource histogram to illustrate how the project can be completed in the minimum time, assuming that each activity is to start as early as possible. *(3 marks)*

(f) Given that there are two workers available for the project, find the minimum completion time for the project. *(1 mark)*

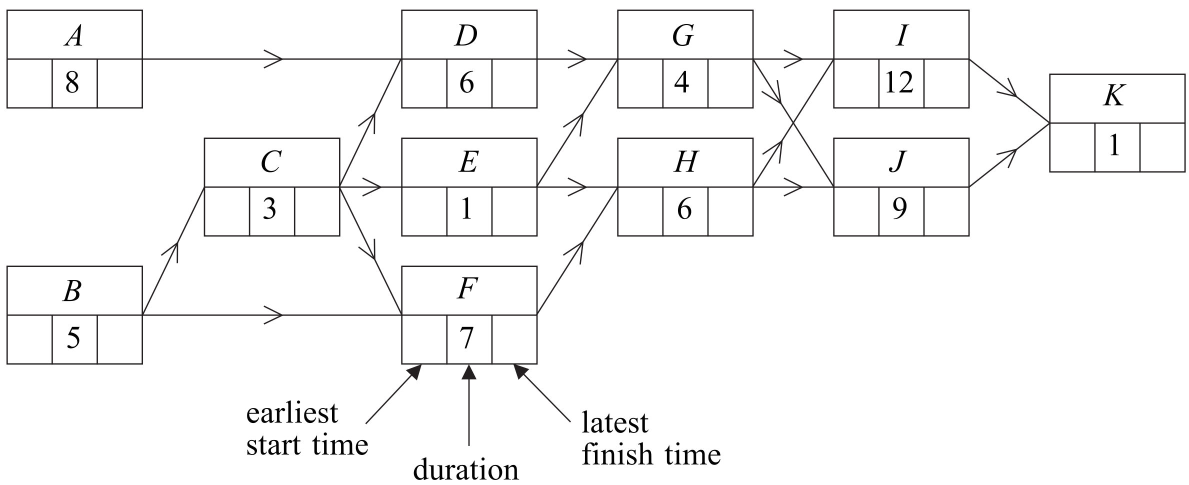
(g) Given that there is only one worker available for the project, find the minimum completion time for the project. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 1

(a)

Figure 1



QUESTION PART REFERENCE

Answer space for question 1

(b)

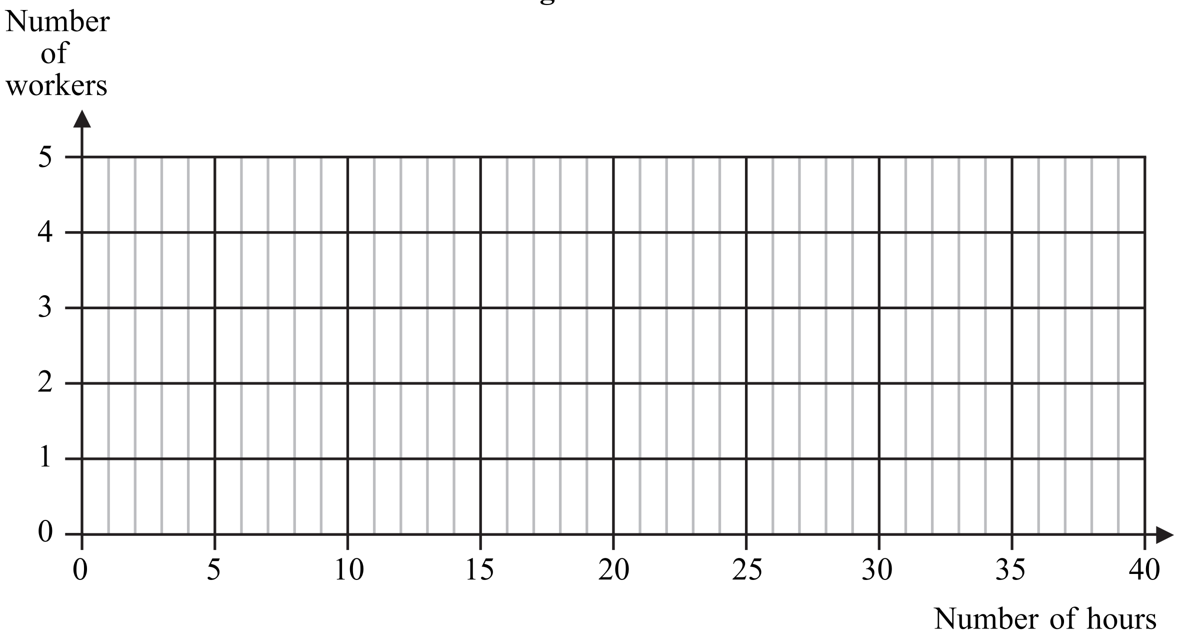
Figure 2

Activity	Immediate predecessor(s)
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	

QUESTION PART REFERENCE

Answer space for question 1

Figure 3



Turn over ▶

2 Harry and Will play a zero-sum game. The game is represented by the following pay-off matrix for Harry.

Will

	Strategy	D	E	F	G
Harry	**A**	4	-1	2	3
	B	4	6	3	7
	C	1	3	-2	4

(a) Show that this game has a stable solution and state the play-safe strategy for each player. *(4 marks)*

(b) List any saddle points. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

3 Four pupils, Wendy, Xiong, Yasmin and Zaira, are each to be allocated a different memory coach from five available coaches: Asif, Bill, Connie, Deidre and Eric. Each pupil has an initial training session with each coach, and a test which scores their improvement in memory-recall produces the following results.

	Asif	Bill	Connie	Deidre	Eric
Wendy	35	38	43	34	37
Xiong	38	37	38	34	36
Yasmin	32	33	31	31	32
Zaira	34	38	35	31	34

(a) Modify the table of results by subtracting each value from 43. *(1 mark)*

(b) Use the Hungarian algorithm, reducing the **rows first**, to assign one coach to one pupil so that the total improvement of the four pupils is maximised.

State the total improvement of the four pupils. *(8 marks)*

QUESTION PART REFERENCE

Answer space for question 3

4 (a) When investigating three network flow problems, a student finds:

(i) a flow of 50 and a cut with capacity 50;

(ii) a flow of 35 and a cut with capacity 50;

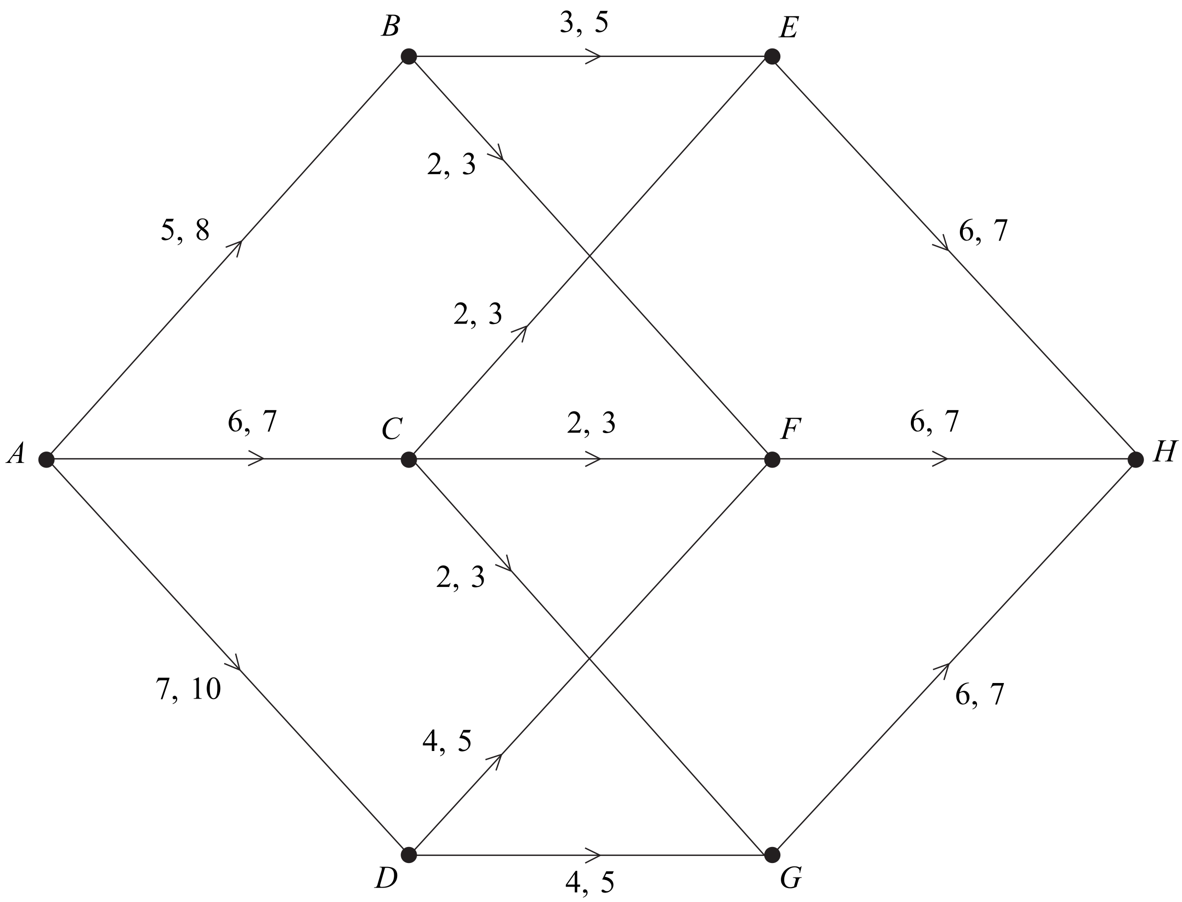
(iii) a flow of 50 and a cut with capacity 35.

In each case, write down what the student can deduce about the maximum flow.

(4 marks)

(b) The diagram below shows a network. The numbers on the arcs represent the minimum and maximum flow along each arc respectively.

By considering the flow at an appropriate vertex, explain why a flow is not possible through this network.



(2 marks)

5 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = x - 2y + 3z$

subject to

$$x + y + z \leqslant 16$$
$$x - 2y + 2z \leqslant 17$$
$$2x - y + 2z \leqslant 19$$

and $x \geqslant 0,\ y \geqslant 0,\ z \geqslant 0$. *(2 marks)*

(b) (i) The first pivot to be chosen is from the z-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

(ii) Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret the tableau that you obtained in part **(c)(i)** and state the values of your slack variables. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

6 Kate and Pippa play a zero-sum game. The game is represented by the following pay-off matrix for Kate.

Pippa

	Strategy	***D***	***E***	***F***
	A	−2	0	3
Kate	***B***	3	−2	−2
	C	4	1	−1

(a) Explain why Kate should not adopt strategy *B*. *(1 mark)*

(b) Find the optimal mixed strategy for Kate and find the value of the game. *(7 marks)*

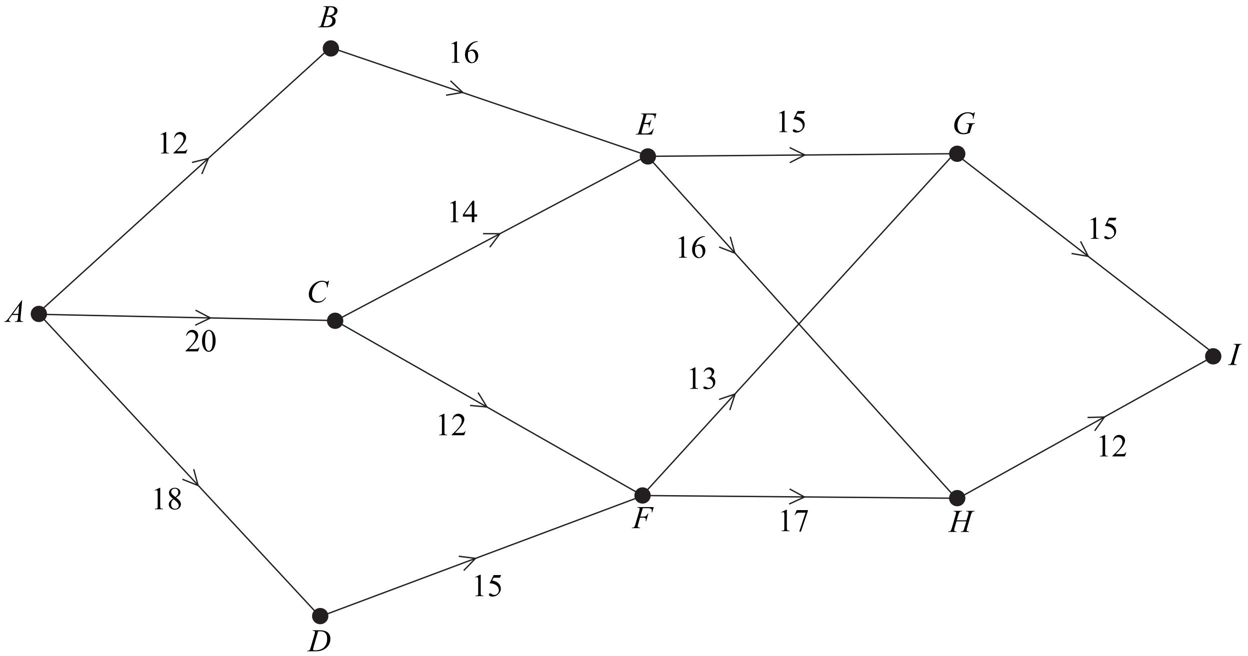
(c) Find the optimal mixed strategy for Pippa. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 6

7 The network below shows a system of one-way roads. The number on each edge represents the number of bags for recycling that can be collected by driving along that road.

A collector is to drive from A to I.



(a) **Working backwards from *I***, use dynamic programming to find the maximum number of bags that can be collected when driving from A to I.

You must complete the table opposite as your solution. *(7 marks)*

(b) State the route that the collector should take in order to collect the maximum number of bags. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 7

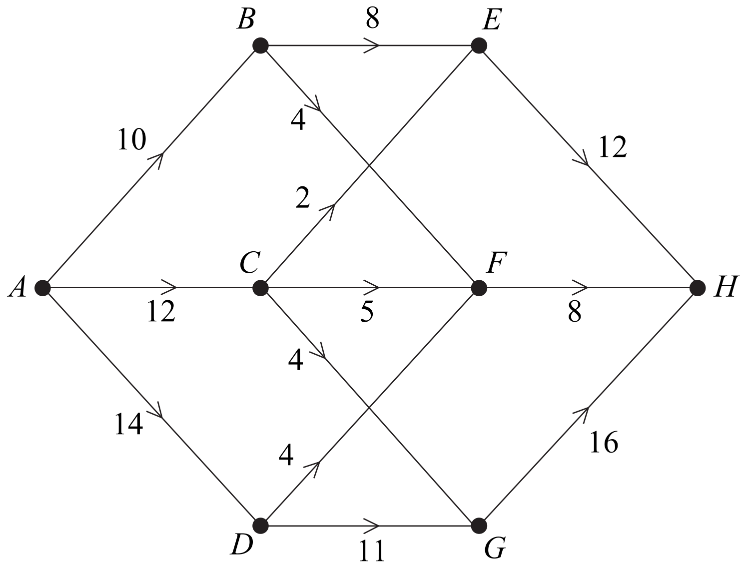
QUESTION PART REFERENCE

Answer space for question 7

(a)

Stage	State	From	Value
1	G	I	
	H	I	
2			

8 The network below represents a system of pipes. The capacity of each pipe, in litres per second, is indicated on the corresponding edge.



(a) Find the maximum flow along each of the routes *ABEH*, *ACFH* and *ADGH* and enter their values in the table on **Figure 4** opposite. *(1 mark)*

(b) (i) Taking your answers to part **(a)** as the initial flow, use the labelling procedure on **Figure 4** to find the maximum flow through the network. You should indicate any flow-augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow and, on **Figure 5** opposite, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut. *(1 mark)*

QUESTION PART REFERENCE	**Answer space for question 8**

QUESTION PART REFERENCE

Answer space for question 8

Route	Flow
ABEH	
ACFH	
ADGH	

Figure 4

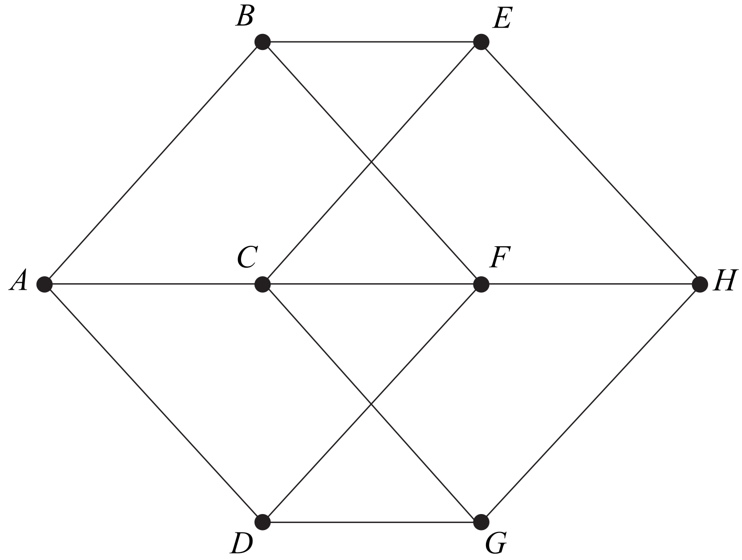
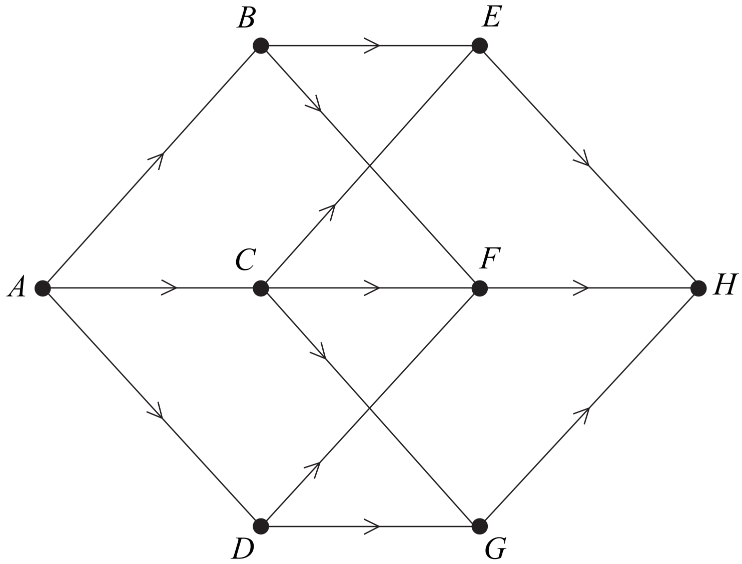


Figure 5



Centre Number						Candidate Number				
Surname										
Other Names										
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Question	Mark
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MD02

Unit Decision 2

Thursday 13 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 1** opposite shows an activity diagram for a project. The duration required for each activity is given in hours. The project is to be completed in the minimum time.

(a) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 1**. *(4 marks)*

(b) Find the critical path. *(1 mark)*

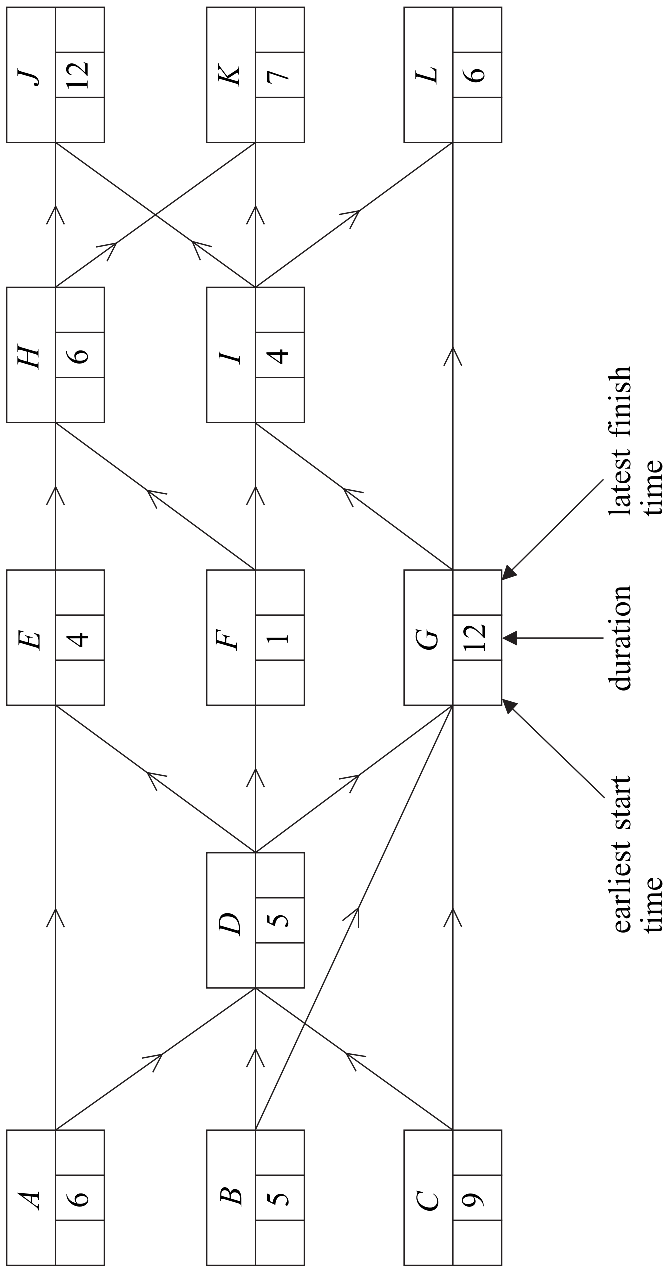
(c) Find the float time of activity E. *(1 mark)*

(d) Given that activities H and K will both overrun by 10 hours, find the new minimum completion time for the project. *(3 marks)*

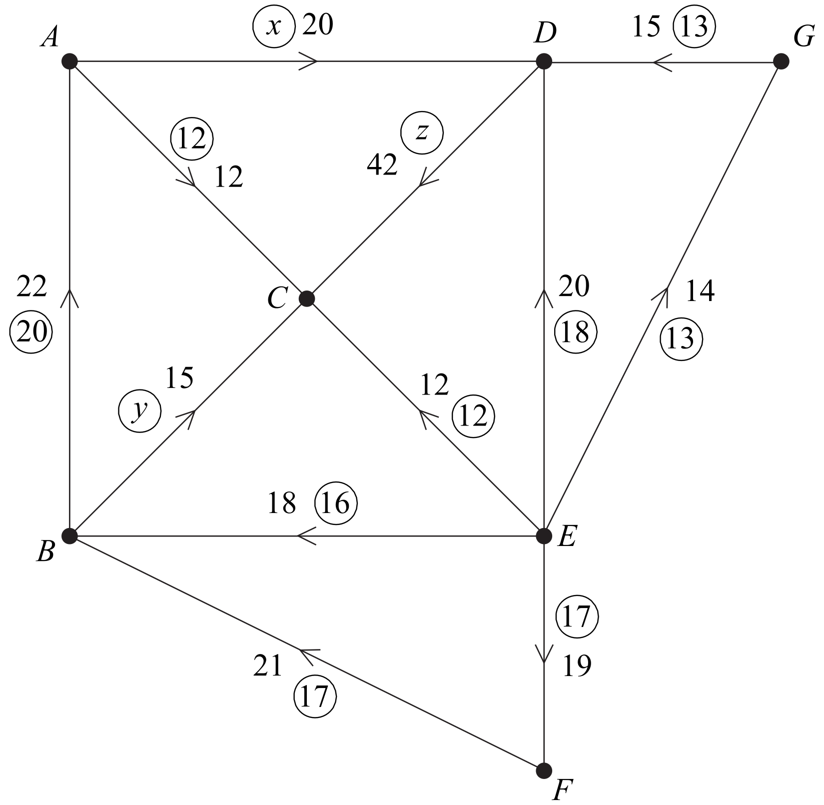
QUESTION PART REFERENCE

Answer space for question 1

Figure 1


A
6
B
5
C
9
D
5
E
4
F
1
G
12
H
6
I
4
J
12
K
7
L
6
earliest start time
duration
latest finish time

2 The network below represents a system of pipes. The number **not** circled on each edge represents the capacity of each pipe in litres per second. The number or letter in each circle represents an initial flow in litres per second.



(a) Write down the capacity of edge EF. *(1 mark)*

(b) State the source vertex. *(1 mark)*

(c) State the sink vertex. *(1 mark)*

(d) Find the values of x, y and z. *(3 marks)*

(e) Find the value of the initial flow. *(1 mark)*

(f) Find the value of a cut through the edges EB, EC, ED, EF and EG. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

....

....

....

....

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QUESTION PART REFERENCE

Answer space for question 2

Turn over ▶

3 The table shows the times taken, in minutes, by five people, A, B, C, D and E, to carry out the tasks V, W, X, Y and Z.

	A	**B**	**C**	**D**	**E**
Task V	100	110	112	102	95
Task W	125	130	110	120	115
Task X	105	110	101	108	120
Task Y	115	115	120	135	110
Task Z	100	98	99	100	102

Each of the five tasks is to be given to a different one of the five people so that the total time for the five tasks is minimised. The Hungarian algorithm is to be used.

(a) By reducing the **columns first**, and then the rows, show that the new table of values is

0	12	13	2	0
14	21	0	k	9
3	10	0	6	23
0	2	6	20	0
0	0	0	0	7

and state the value of the constant k. *(3 marks)*

(b) Show that the zeros in the table in part **(a)** can be covered with four lines. Use augmentation **twice** to produce a table where five lines are required to cover the zeros. *(5 marks)*

(c) Hence find the possible ways of allocating the five tasks to the five people to achieve the minimum total time. *(3 marks)*

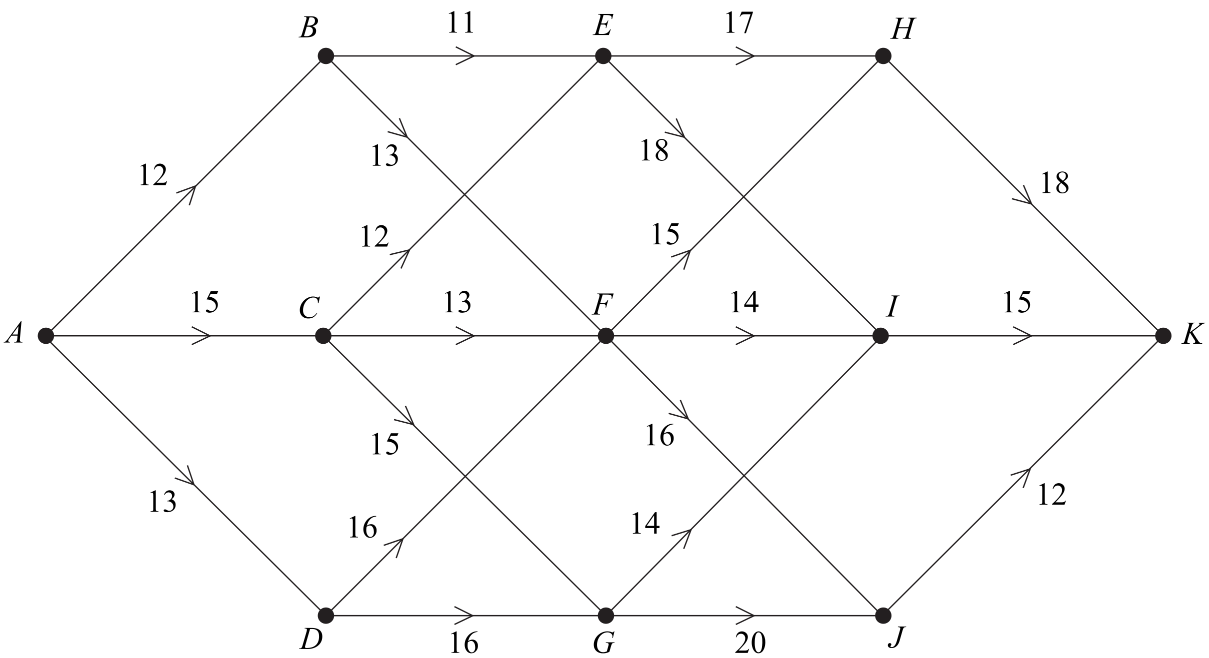
(d) Find the minimum total time. *(1 mark)*

QUESTION PART REFERENCE	**Answer space for question 3**

4 A haulage company, based in town A, is to deliver a tall statue to town K. The statue is being delivered on the back of a lorry.

The network below shows a system of roads. The number on each edge represents the height, in feet, of the lowest bridge on that road.

The company wants to ensure that the height of the lowest bridge along the route from A to K is maximised.



Working backwards from K, use dynamic programming to find the optimal route when driving from A to K.

You must complete the table opposite as your solution. *(9 marks)*

QUESTION PART REFERENCE

Answer space for question 4

Stage	State	From	Value
1	H	K	
	I	K	
	J	K	
2			

Optimal route is ....

5 Romeo and Juliet play a zero-sum game. The game is represented by the following pay-off matrix for Romeo.

Juliet

	Strategy	**D**	**E**	**F**
	A	4	−4	0
Romeo	**B**	−2	−5	3
	C	2	1	−2

(a) Find the play-safe strategy for each player. *(3 marks)*

(b) Show that there is no stable solution. *(1 mark)*

(c) Explain why Juliet should never play strategy D. *(1 mark)*

(d) (i) Explain why the following is a suitable pay-off matrix **for Juliet**.

4	5	−1
0	−3	2

(2 marks)

(ii) Hence find the optimal strategy for Juliet. *(7 marks)*

(iii) Find the value of the game for Juliet. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 5

6 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 4x + 3y + z$

subject to
$$2x + y + z \leqslant 25$$
$$x + 2y + z \leqslant 40$$
$$x + y + 2z \leqslant 30$$

and $x \geqslant 0,\ \ y \geqslant 0,\ \ z \geqslant 0\,.$ *(2 marks)*

(b) The first pivot to be chosen is from the x-column.

Perform one iteration of the Simplex method. *(3 marks)*

(c) (i) Perform one further iteration. *(3 marks)*

(ii) Interpret your final tableau and state the values of your slack variables. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 6

7 **Figure 2** shows a network of pipes.

Water from two reservoirs, R_1 and R_2, is used to supply three towns, T_1, T_2 and T_3.

In **Figure 2**, the capacity of each pipe is given by the number **not** circled on each edge. The numbers in circles represent an initial flow.

(a) Add a supersource, supersink and appropriate weighted edges to **Figure 2**. *(2 marks)*

(b) (i) Use the initial flow and the labelling procedure on **Figure 3** to find the maximum flow through the network.

You should indicate any flow augmenting routes in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(ii) State the value of the maximum flow and, on **Figure 4**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut. *(2 marks)*

Figure 2

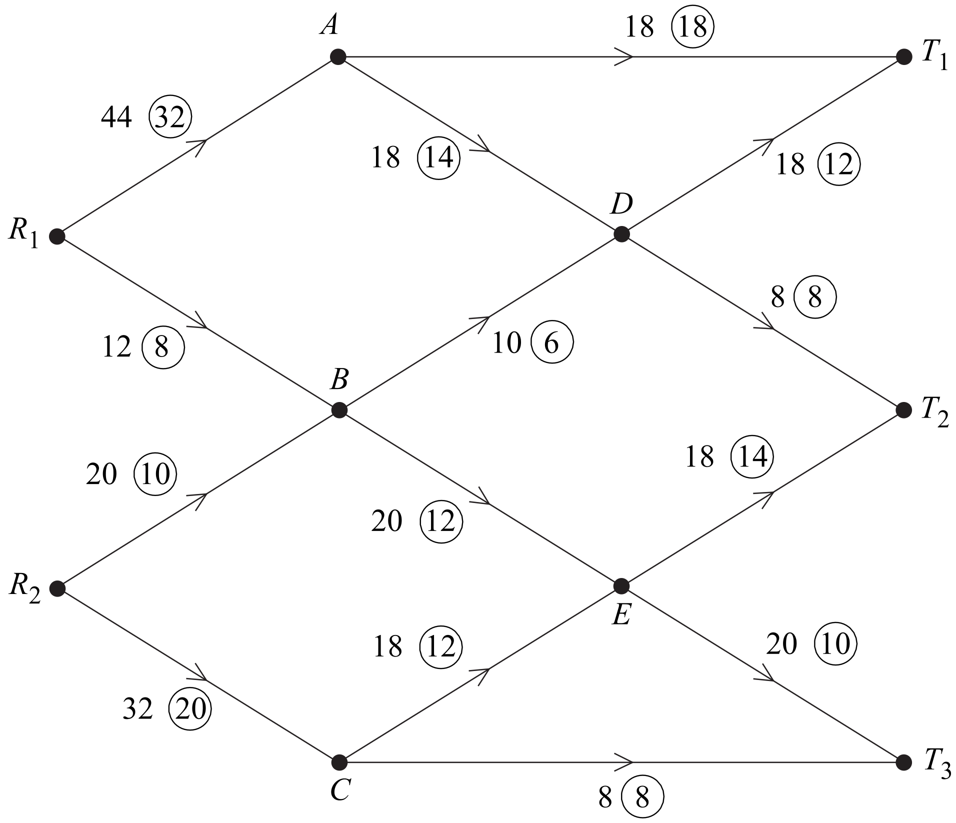
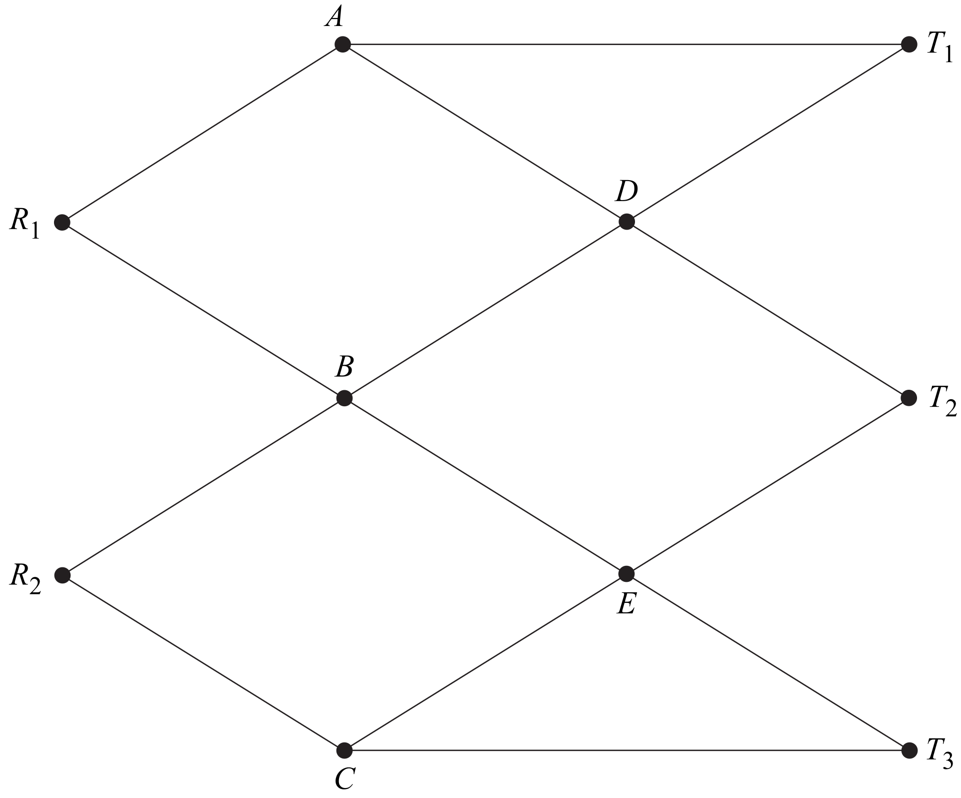
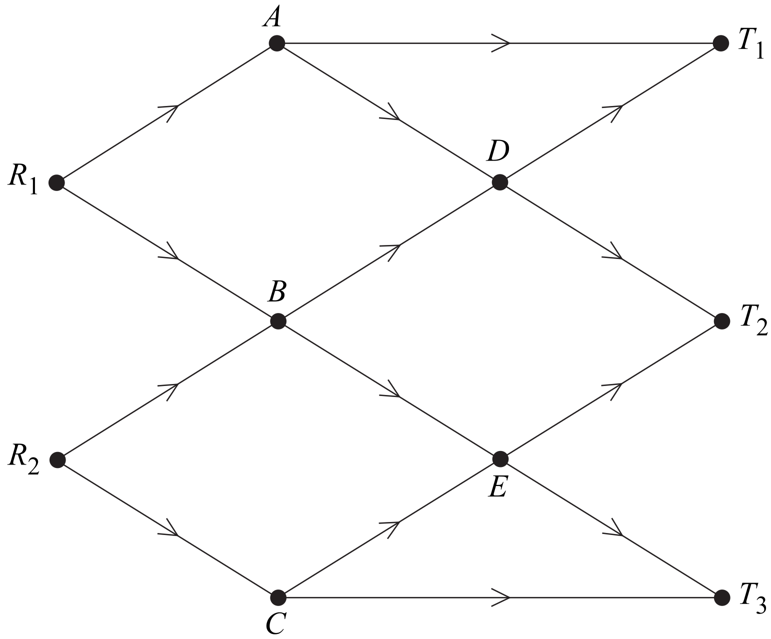


Figure 3



Route	Flow

Figure 4



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AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MD02

Unit Decision 2

Tuesday 24 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
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- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A major project has been divided into a number of tasks, as shown in the table. The minimum time required to complete each task is also shown.

Activity	**Immediate predecessor**	**Duration (hours)**
A	–	3
B	A	3
C	A	4
D	B, C	6
E	B, C	5
F	C	2
G	C	1
H	A	15
I	D, E	4
J	F	6
K	G	10
L	H, I, J, K	1

(a) On the page opposite, construct an activity network for the project. (Activity A has already been drawn.)

[3 marks]

(b) Find the earliest start time for each activity.

[2 marks]

(c) Find the latest finish time for each activity.

[2 marks]

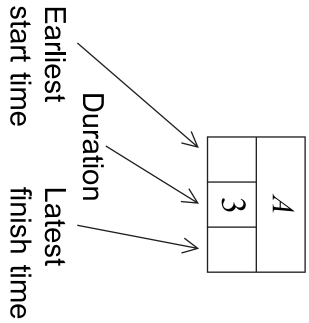
(d) List the critical activities.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

Answer space for question 1



2 Alex and Roberto play a zero-sum game. The game is represented by the following pay-off matrix for Alex.

Roberto

	Strategy	**D**	**E**	**F**	**G**
	A	5	-4	-1	1
Alex	**B**	4	3	0	1
	C	-3	0	-5	-2

(a) Show that this game has a stable solution and state the play-safe strategy for each player.

[4 marks]

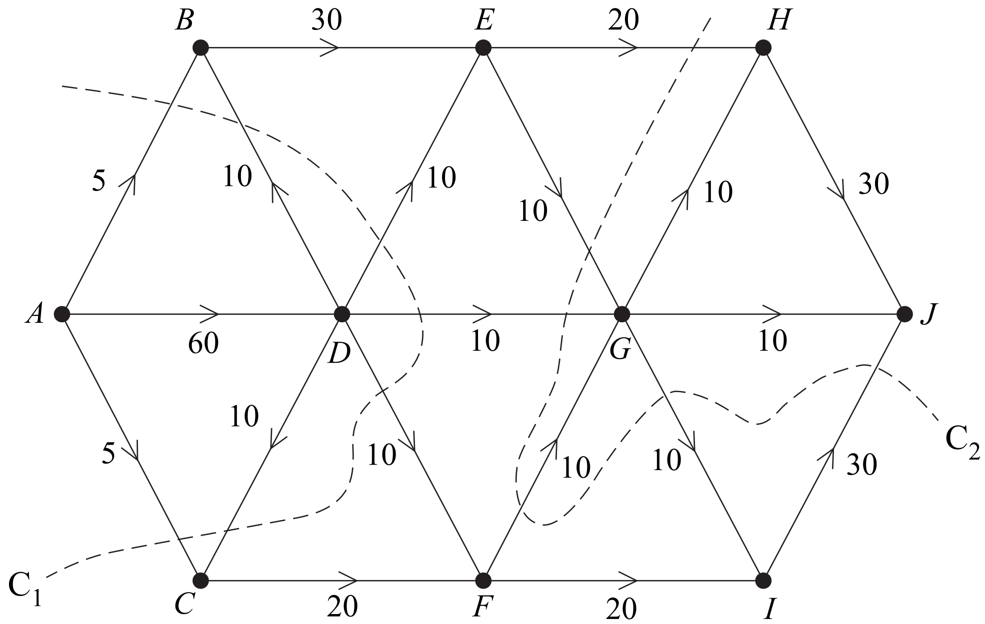
(b) List any saddle points.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 2

3 The diagram below shows a network of pipes with source A and sink J. The capacity of each pipe is given by the number on each edge.



(a) Find the values of the cuts C_1 and C_2.

[2 marks]

(b) Find by inspection a flow of 60 units, with flows of 25, 10 and 25 along HJ, GJ and IJ respectively. Illustrate your answer on **Figure 1**.

[2 marks]

(c) (i) On a certain day the section EH is blocked, as shown on **Figure 2**.

Find, by inspection or otherwise, the maximum flow on this day and illustrate your answer on **Figure 2**.

[3 marks]

(ii) Show that the flow obtained in part **(c)(i)** is maximal.

[2 marks]

QUESTION PART REFERENCE

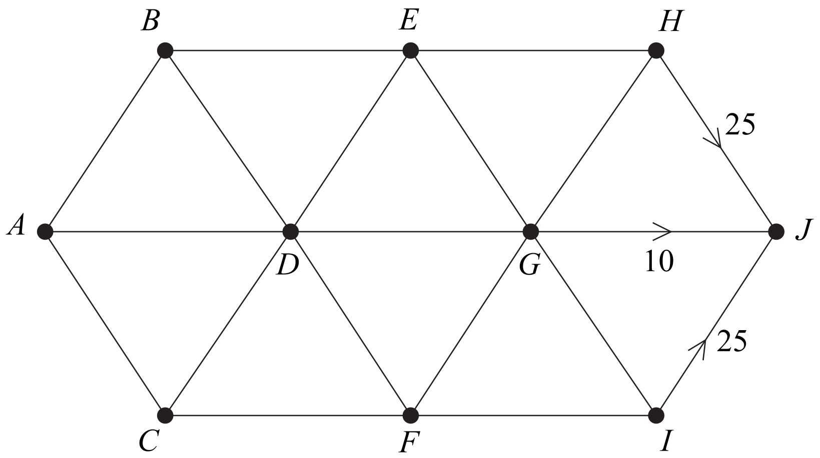
Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

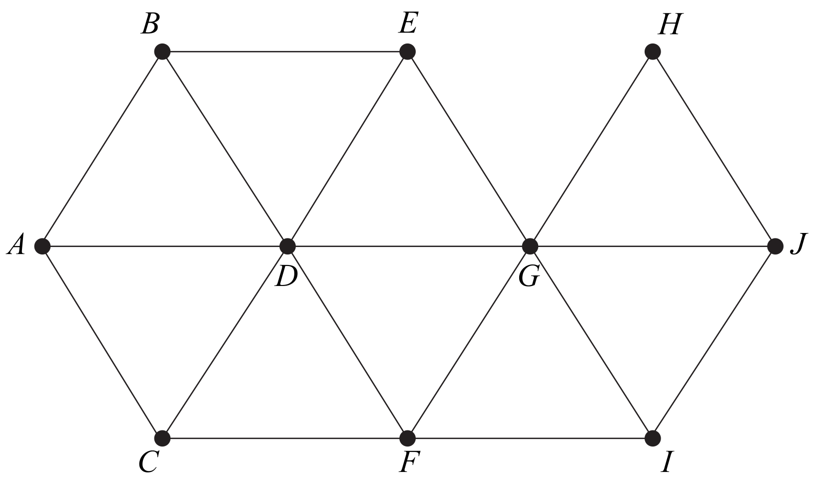
(b)

Figure 1



(c)

Figure 2



Maximum flow = ______________________

4 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 3x + 6y + 2z$

subject to $x + 3y + 2z \leqslant 11$

$3x + 4y + 2z \leqslant 21$

and $x \geqslant 0\ ,\ \ y \geqslant 0\ ,\ \ z \geqslant 0\ .$

[2 marks]

(b) The first pivot to be chosen is from the y-column.

Perform one iteration of the Simplex method.

[3 marks]

(c) Perform one further iteration.

[3 marks]

(d) Interpret the tableau obtained in part **(c)** and state the values of your slack variables.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4

5 Mark and Owen play a zero-sum game. The game is represented by the following pay-off matrix for Mark.

Owen

	Strategy	**D**	**E**	**F**
	A	4	1	-1
Mark	**B**	3	-2	-2
	C	-2	0	3

(a) Explain why Mark should never play strategy B.

[1 mark]

(b) It is given that the value of the game is 0.6. Find the optimal strategy for **Owen**.

(You are **not** required to find the optimal mixed strategy for Mark.)

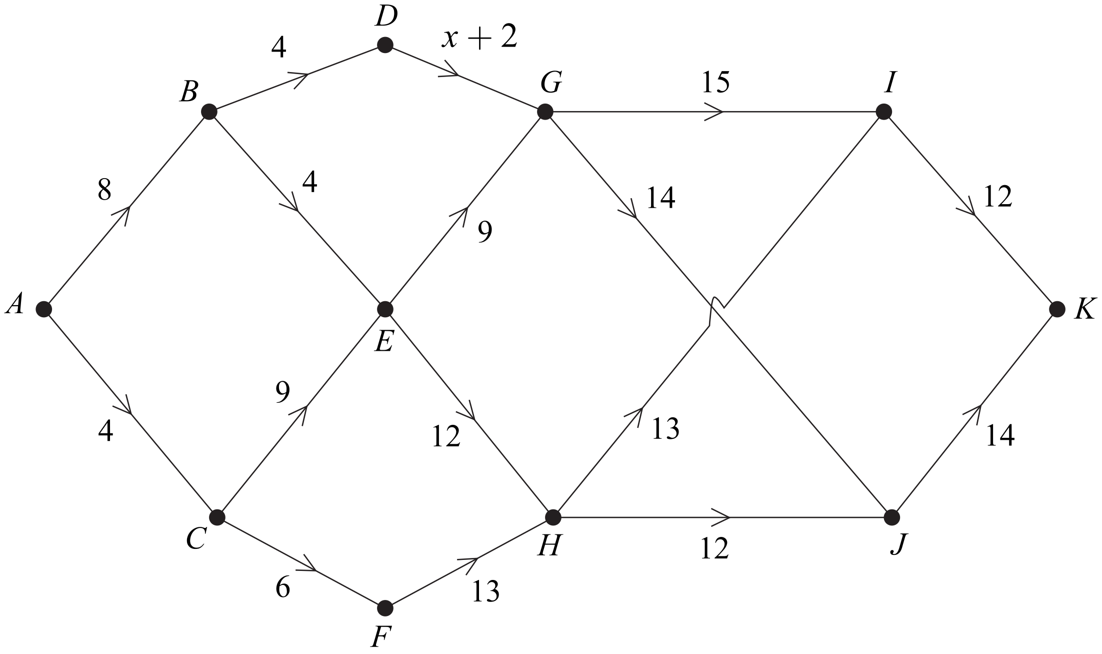
[7 marks]

QUESTION PART REFERENCE

Answer space for question 5

6 The network below has 11 vertices and 16 edges connecting some pairs of vertices. The numbers on the edges are their weights. The weight of the edge DG is given in terms of x.

There are three routes from A to K that have the same minimum total weight.



Working backwards from K, use dynamic programming, to find:

(a) the minimum total weight from A to K;

(b) the value of x;

(c) the three routes corresponding to the minimum total weight.

You must complete the table opposite as your solution.

[12 marks]

QUESTION PART REFERENCE

Answer space for question 6

Answer space for question 6

Stage	State	From	Calculation	Value
1	I	K		
	J	K		

7 The table shows the times taken, in minutes, by four people, A, B, C and D, to carry out the tasks W, X, Y and Z.

Some of the times are subject to the same delay of x minutes, where $4 < x < 11$.

	A	***B***	***C***	***D***
Task *W*	$x+8$	$x+4$	$x+6$	$x+9$
Task *X*	$x+5$	$x+3$	$x+4$	$x+2$
Task *Y*	$x+8$	$x+7$	$x+5$	$2x+2$
Task *Z*	$x+3$	$2x-3$	12	$x+1$

Each of the four tasks is to be given to a different one of the four people so that the total time for the four tasks is minimised.

(a) The minimum time to complete task Z is $(x+1)$.

Write down the minimum time to complete task W, task X and task Y.

[2 marks]

(b) Use the Hungarian algorithm, by reducing the **rows** first, to assign each task to a different person so that the total time for the four tasks is minimised.

[7 marks]

(c) Given that the minimum total time is 42 minutes, find the value of x.

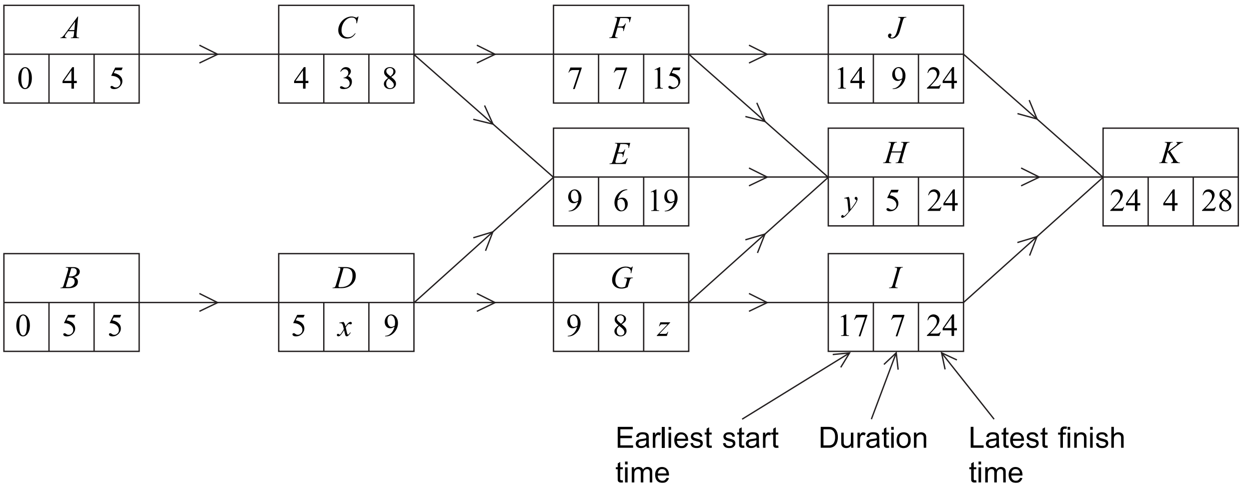
[2 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

8 An activity diagram for a project is shown below. The duration of each activity is given in weeks. The earliest start time and the latest finish time for each activity are shown on the diagram.



(a) Find the values of x, y and z.

[2 marks]

(b) State the critical path.

[1 mark]

(c) Some of the activities can be speeded up at an additional cost. The following table lists the activities that can be speeded up together with the minimum possible duration of these activities. The table also shows the additional cost of reducing the duration of each of these activities by one week.

Activity	Additional cost per week (£)	Minimum completion time (weeks)
E	8000	1
F	7000	4
G	6000	5

The company wishes to complete the project as soon as possible.

(i) Find which activities should be speeded up. For **each** such activity, state, with justification, the reduction in the number of weeks.

(ii) Hence state the revised minimum time for the completion of the whole project.

(iii) Calculate the total additional cost that the company would incur in meeting this revised completion time.

[7 marks]

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

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TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MD02

Unit Decision 2

Wednesday 24 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
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- Fill in the boxes at the top of this page.
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- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 **Figure 2**, on the page opposite, shows an activity diagram for a project. Each activity requires one worker. The duration required for each activity is given in hours.

(a) On **Figure 1** below, complete the precedence table.

[1 mark]

(b) Find the earliest start time and the latest finish time for each activity and insert their values on **Figure 2**.

[4 marks]

(c) List the critical paths.

[2 marks]

(d) Find the float time of activity E.

[1 mark]

(e) Using **Figure 3** opposite, draw a Gantt diagram to illustrate how the project can be completed in the minimum time, assuming that each activity is to start as early as possible.

[3 marks]

(f) Given that there is only one worker available for the project, find the minimum completion time for the project.

[1 mark]

(g) Given that there are two workers available for the project, find the minimum completion time for the project. Show a suitable allocation of tasks to the two workers.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

Figure 1

Activity	Immediate predecessor(s)
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 1

Figure 2

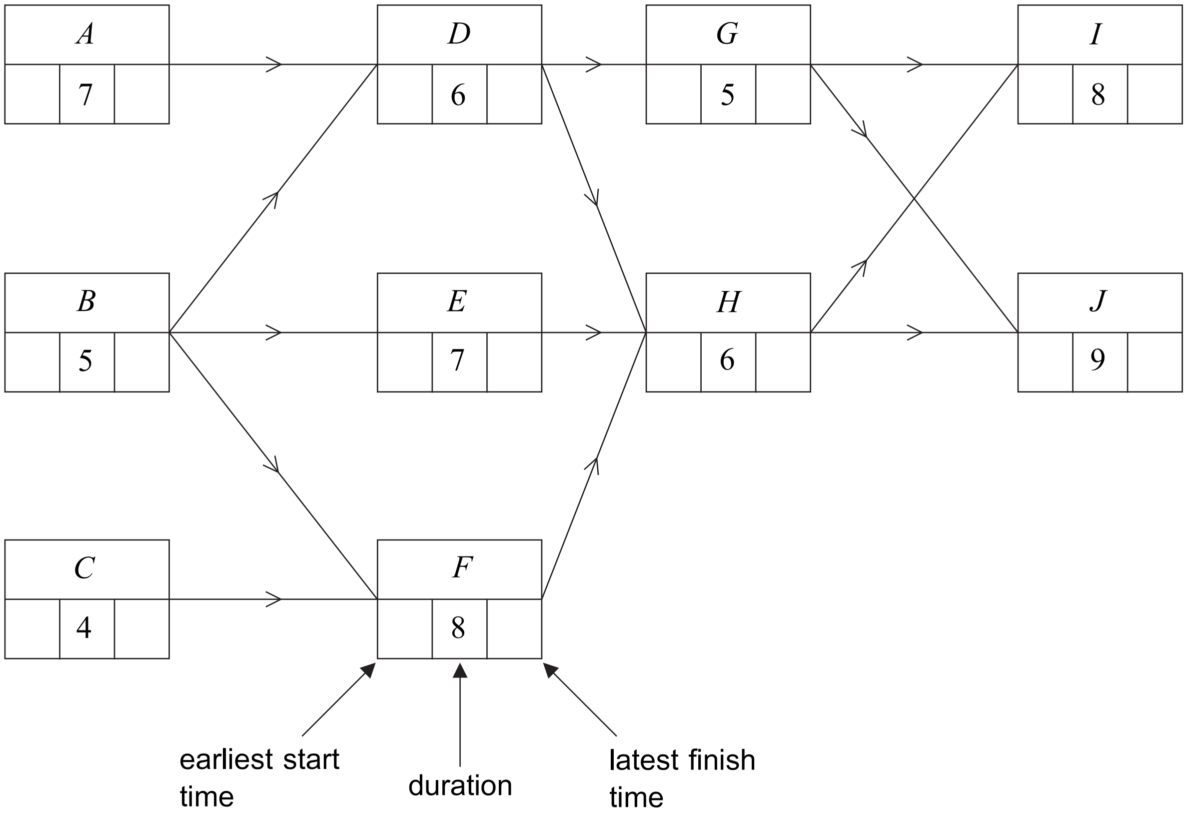
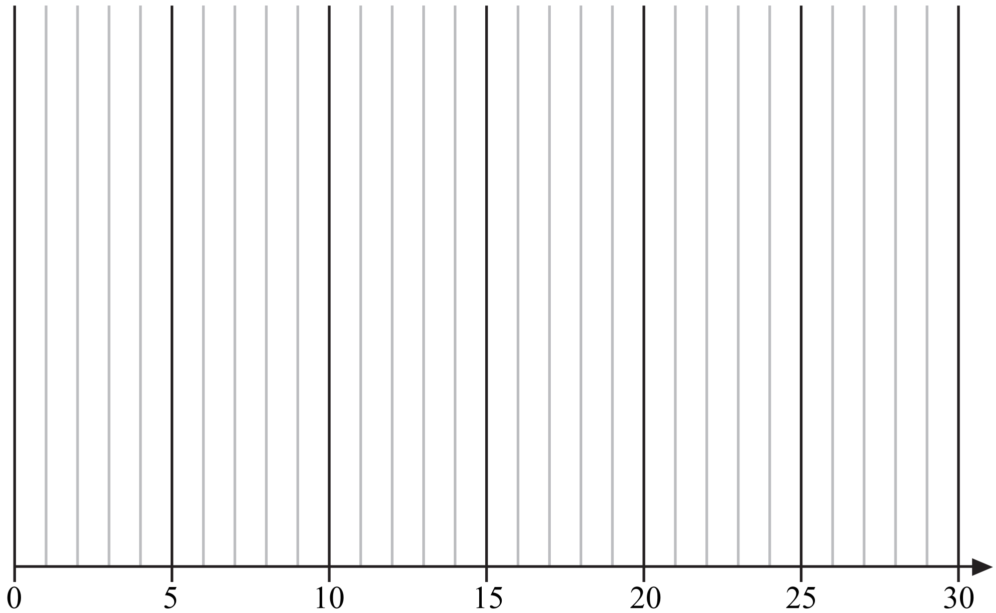


Figure 3



2 Stan and Christine play a zero-sum game. The game is represented by the following pay-off matrix for Stan.

Christine

	Strategy	D	E	F	G
Stan	**A**	3	−3	−1	0
	B	−1	−4	2	3
	C	1	0	−3	−2

(a) Find the play-safe strategy for each player.

[3 marks]

(b) Show that there is no stable solution.

[1 mark]

(c) Explain why a suitable pay-off matrix **for Christine** is given by

3	4	0
1	−2	3

[4 marks]

QUESTION PART REFERENCE

Answer space for question 2

3 In the London 2012 Olympics, the Jamaican 4×100 metres relay team set a world record time of 36.84 seconds.

Athletes take different times to run each of the four legs.

The coach of a national athletics team has five athletes available for a major championship. The lowest times that the five athletes take to cover each of the four legs is given in the table below.

The coach is to allocate a different athlete from the five available athletes, A, B, C, D and E, to each of the four legs to produce the lowest total time.

	Leg 1	**Leg 2**	**Leg 3**	**Leg 4**
Athlete *A*	9.84	8.91	8.98	8.70
Athlete *B*	10.28	9.06	9.24	9.05
Athlete *C*	10.31	9.11	9.22	9.18
Athlete *D*	10.04	9.07	9.19	9.01
Athlete *E*	9.91	8.95	9.09	8.74

Use the Hungarian algorithm, by reducing the **columns first**, to assign an athlete to each leg so that the total time of the four athletes is minimised.

State the allocation of the athletes to the four legs and the total time.

[11 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 (a) Display the following linear programming problem in a Simplex tableau.

Maximise $P = 2x + 3y + 4z$

subject to
$$x + y + 2z \leqslant 20$$
$$3x + 2y + z \leqslant 30$$
$$2x + 3y + z \leqslant 40$$

and $x \geqslant 0\ ,\ y \geqslant 0\ ,\ z \geqslant 0$

[2 marks]

(b) (i) The first pivot to be chosen is from the z-column. Identify the pivot and explain why this particular value is chosen.

[2 marks]

(ii) Perform one iteration of the Simplex method.

[3 marks]

(c) (i) Perform one further iteration.

[3 marks]

(ii) Interpret your final tableau and state the values of your slack variables.

[3 marks]

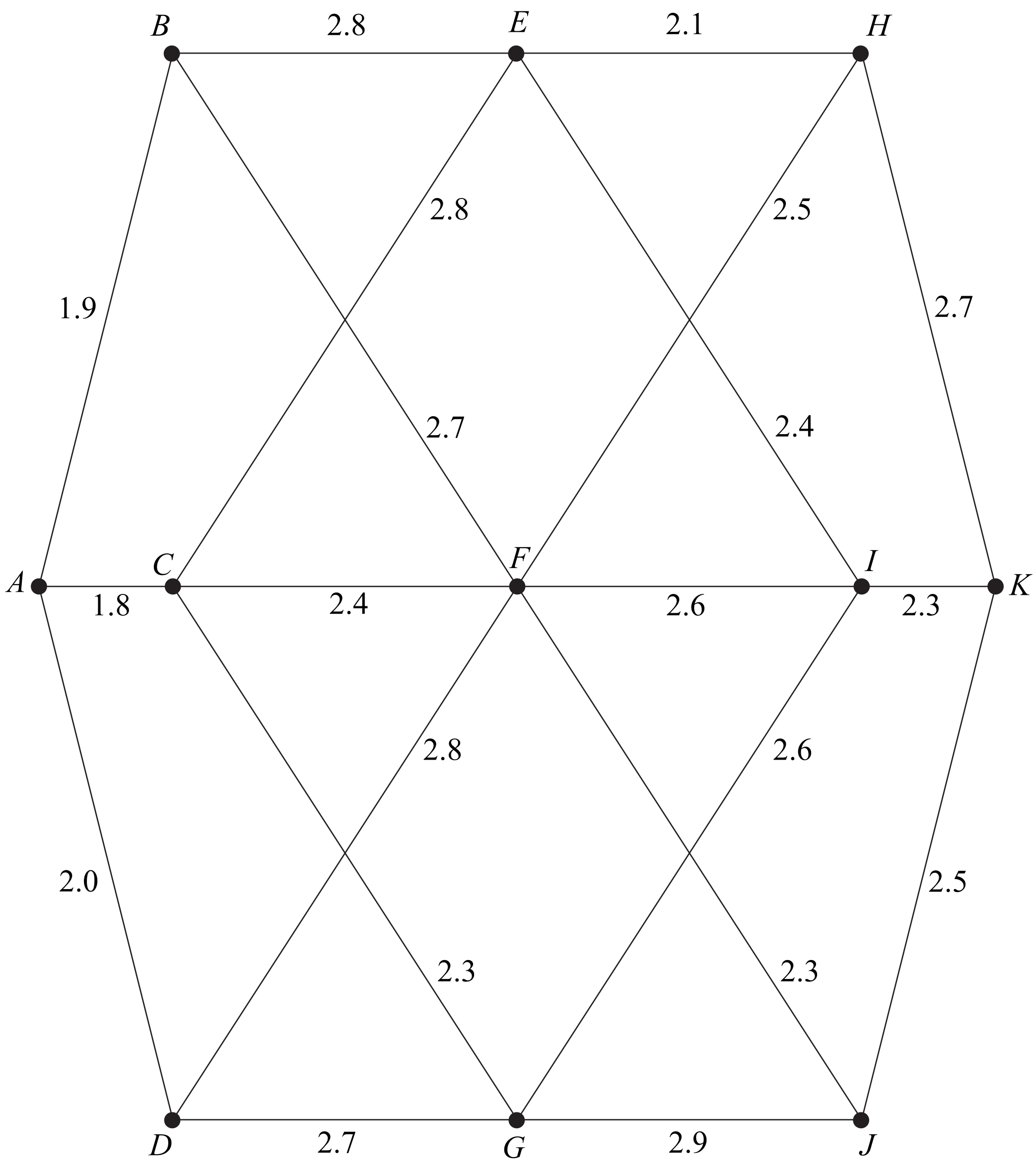
QUESTION PART REFERENCE

Answer space for question 4

5 Tom is going on a driving holiday and wishes to drive from A to K.

The network below shows a system of roads. The number on each edge represents the maximum altitude of the road, in hundreds of metres above sea level.

Tom wants to ensure that the maximum altitude of any road along the route from A to K is minimised.



(a) **Working backwards from K**, use dynamic programming to find the optimal route when driving from A to K.

You must complete the table opposite as your solution.

[9 marks]

(b) Tom finds that the road CF is blocked. Find Tom's new optimal route and the maximum altitude of any road on this route.

[2 marks]

Answer space for question 5

Stage	State	From	Value
1	H	K	
	I	K	
	J	K	
2			

(a) Optimal route is ....

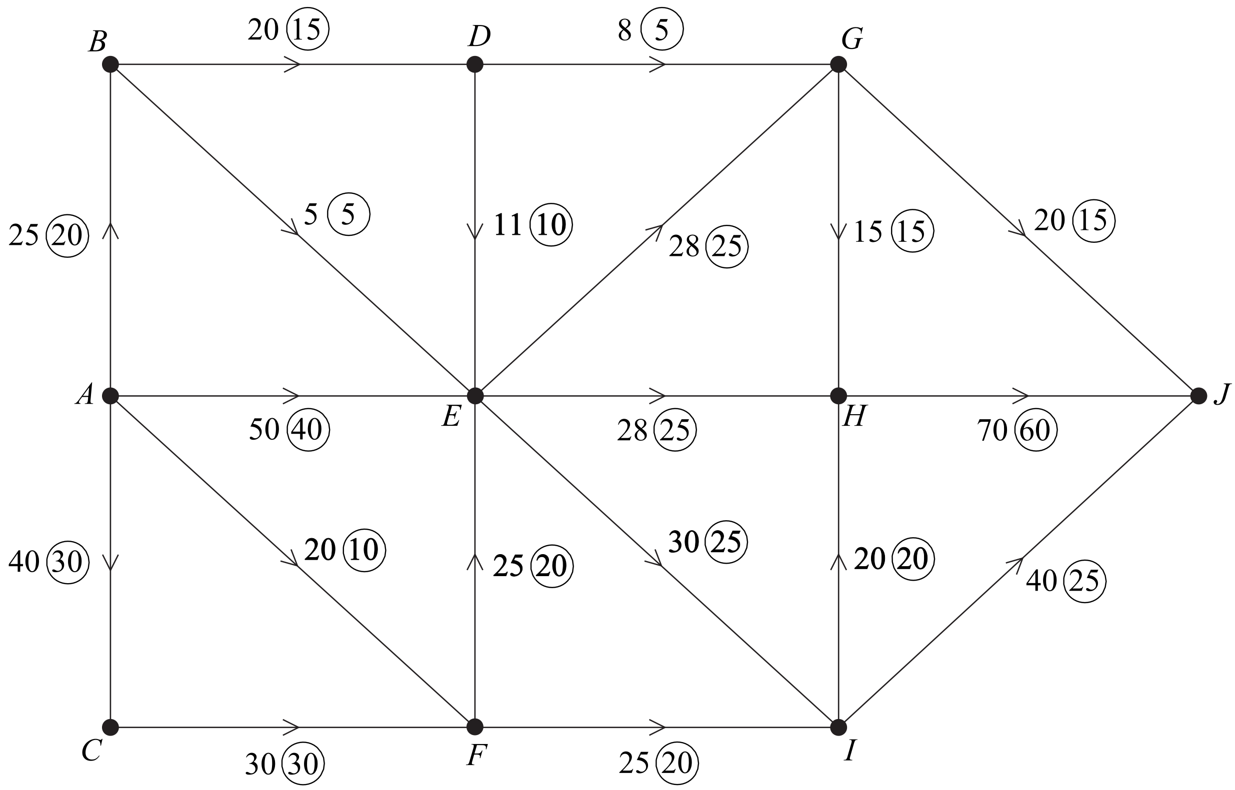
(b) Tom's route is ....

Maximum altitude is ....

6 **Figure 4** below shows a network of pipes.

The capacity of each pipe is given by the number **not circled** on each edge.
The numbers in circles represent an initial flow.

Figure 4



(a) Find the value of the initial flow.

[1 mark]

(b) (i) Use the initial flow and the labelling procedure on **Figure 5** to find the maximum flow through the network. You should indicate any flow-augmenting routes in the table and modify the potential increases and decreases of the flow on the network.

[5 marks]

(ii) State the value of the maximum flow and, on **Figure 6**, illustrate a possible flow along each edge corresponding to this maximum flow.

[2 marks]

(c) Confirm that you have a maximum flow by finding a cut of the same value. List the edges of your cut.

[2 marks]

(d) On a particular day, there is a restriction at vertex G which allows a maximum flow through G of 30.

Find, by inspection, the maximum flow through the network on this day.

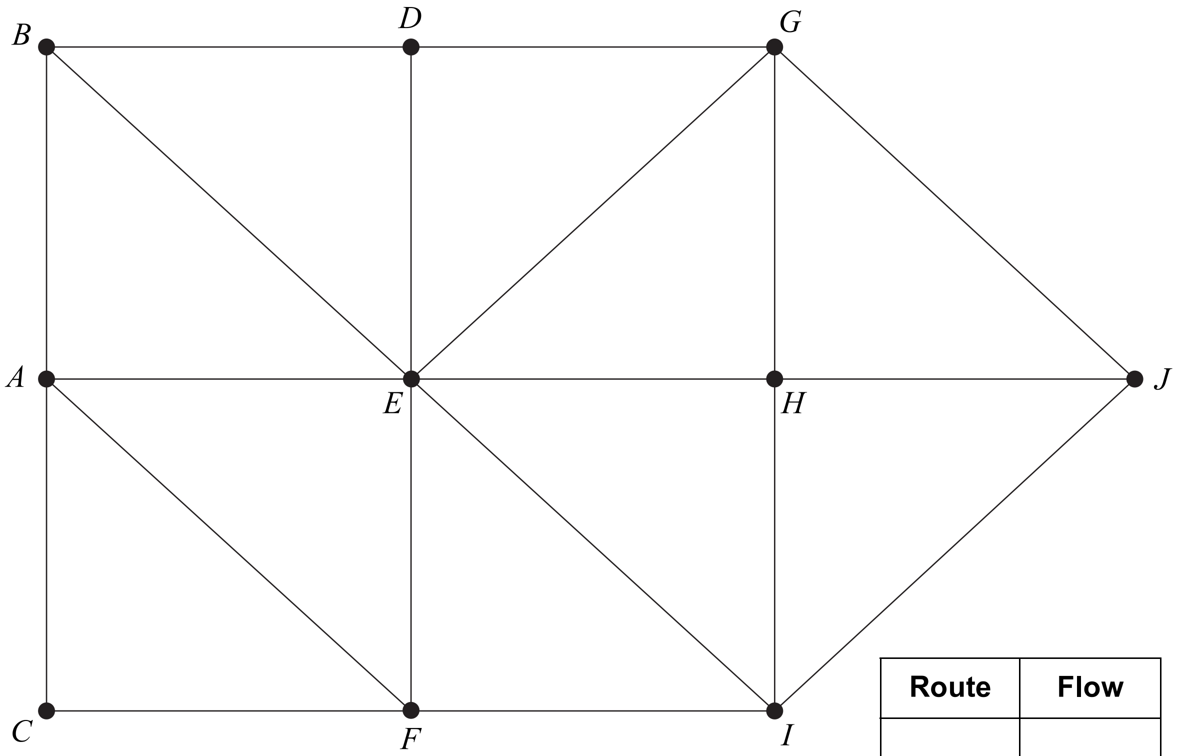
[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

(a) Initial flow = ...

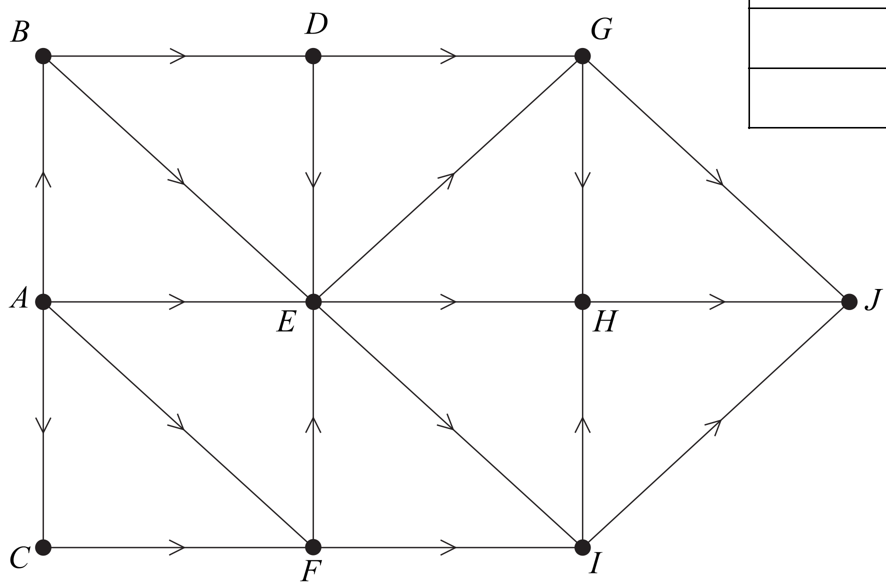
(b)(i) **Figure 5**



Route	Flow

(b)(ii) Maximum flow = ...

Figure 6



7 Arsene and Jose play a zero-sum game. The game is represented by the following pay-off matrix for Arsene, where x is a constant.

The value of the game is 2.5.

		Jose	
	Strategy	**C**	**D**
Arsene	**A**	$x+3$	1
	B	$x+1$	3

(a) Find the optimal mixed strategy for Arsene.

[4 marks]

(b) Find the value of x.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 7